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WORKING PAPER 2007:4

RAPPORT
2007-09-13

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by

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* I wish to thank professor Wlodek Rabinowicz for constructive comments. This study has been funded by the Swedish Council for Working Life and Social Research.

**Abstract**

In this paper we analyze a number of assumptions and conceptual issues that arise in applications of conventional job evaluations, which are used in order to implement the principle “Equal Pay for Jobs of Equal Value” according to the Equal Pay Acts.

The main findings of the analysis can be summarized as follows: 1) A lack of a distinction between subjective and objective criteria as well as between descriptive and evaluative criteria, 2) A defective interpretation of independency conditions that are necessary in order to represent evaluation of jobs by weighted sums of scores, 3) An incorrect diagnosis and subsequently incorrect remedies of defects in job evaluation methods, 4) An incorrect interpretation of the meaning of key concepts such as “Jobs of Equal Value”, 5) Unwarranted assumptions about formal features of relations defined by the concept “Jobs of Equal Value”.

## 1. Introduction

The principle "Equal Pay for Jobs of Equal Value" is an important starting point for arguing the occurrence of gender wage discrimination. The principle is also codified in the Equal Pay Acts for member countries of European union. The concept "Jobs of Equal Value" thus plays a vital role in the argumentation for the occurrences of wage discrimination by gender.[1]

The claim that two jobs are of equal value is based on an evaluative comparison of jobs with respect to demands and difficulties. In most Equal Pay Acts the demand and difficulties that should be considered are stated as four main criteria: *Skills, Responsibility, Effort and Working Conditions*. In various practical applications these criteria are divided into a number of sub-criteria or factors, which constitute the basis for an evaluative comparisons of the jobs. In order to make the evaluative comparison tractable, job evaluation methods are applied.

Needless to say, a job evaluation process is complex in nature and is based on a number of assumptions and concepts of both formal and normative characteristics that can be questioned. It is of course important that such assumptions and conceptual issues are identified and well understood in order to properly evaluate argumentation for occurrence of gender wage discrimination.

The purpose of the paper is to identify and analyze some of these assumptions and conceptual issues that arise in applications of conventional job evaluation methods and are used in order to implement the principle "Equal Pay for Jobs of Equal Value". For this purpose we start by constructing a formal framework. Based on the formal framework we discuss the following issues:

1) A lack of a distinction between subjective and objective criteria as well as between descriptive and evaluative criteria.
2) A defective interpretation of independency conditions that are necessary in order to represent evaluation of jobs by weighted sum of scores.
3) An incorrect diagnosis and subsequently incorrect remedies for defects in job evaluation methods.
4) An incorrect interpretation of the meaning and functioning of key concepts such as "Jobs of Equal Value".
5) Unwarranted assumptions about formal features of relations defined by the concept "Jobs of Equal Value".

[1] See Council Directive 75/117/EEC of 10 February 1975 on the approximation of the laws of the Member States relating to the application of the principle of equal pay for men and women.

The formal framework and subsequent analyses carried out in the paper is based on theories and concepts developed in such disciplines as Multi-Criteria Decision Analysis, Aggregation Theory as applied in "Social Choice Theory", Applied Ethics and Labour Economics.[2]

The paper is organized as follows. In the second section we give a short presentation of a job evaluation system applied in an equal pay project supported by the European Commission. In the third section we construct a formal framework for the analysis of job evaluations. In the fourth section we discuss independency conditions, which are important presumptions in job evaluations. In the fifth section we discuss diagnosis and remedies of possible defects in job evaluation models. In the sixth section we give an analysis of the meaning and functioning of evaluative concepts such as "of equal value" and "of more value". In the seventh section we give an analysis of the formal features of relationships such as "of equal value" and "of more value". In the eighth section we summarize the paper.

## 2. A conventional and representative job evaluation system

We give a short description of a job evaluation system, named *Steps to Pay Equity,* applied in the European Project on Equal Pay, which is supported by European Commission (Harriman and Holm 2001). We assume that the system and its way to use numerical information is representative for many job evaluation systems, the purpose of which is to reveal indication of a gender biased pay structure.

The system *Steps to Pay Equity* can briefly be described as follows. Eight criteria or factors are recommended as grounds for an evaluative comparison of a set of jobs. Each factor is divided as default into five levels, which are scored from 1 to 5. Definitions of the factors are in the appendix. Each factor is assigned a weight in percent, which intends to express the importance of the factor according to the user. Henceforth we use the term decision maker (DM) to refer to users or to persons responsible for the evaluation. The job evaluation process starts with establishing job descriptions of all jobs, which then serve as basic information in the job evaluation process. Each job is then classified with one of the defined levels for each factor, which the DM judges to best fit the job description. In a final step all factors are assigned weights in percent, which means that 100 percent is distributed among the factors

[2] Important references related to the four disciplines are Keeney and Raiffa (1993), Arrow (1963), Sen (1970), Lazear (1998), Hare (1989) and Killingsworth (1990).

according to the DM's assessment of the relative importance of the factors. An example of assigning weights to the eight criteria is presented in Figure 1.

**Figure 1**: Factors and weights

| Factors | Weight % |
|---|---|
| SKILL | 50 |
| 1. Education/experience | 20 |
| 2. Problem solving | 15 |
| 3. Social skills | 15 |
| RESPONSIBILITY FOR | 40 |
| 4. material resources and information | 10 |
| 5. people | 10 |
| 6. planning, development, results, work management | 20 |
| WORKING CONDITIONS | 10 |
| 7. Physical conditions | 5 |
| 8. Mental conditions | 5 |

*Source: Steps to Pay Equity,* see Harriman and Holm (2001).

Based on the classification of jobs on the defined levels for each factor, each job is then assigned a total score in terms of a weighted sum of scores, which represent the evaluation of jobs with respect to (w. r. t.) each factor.

Thus in *Steps to Pay Equity* as well as in most job evaluation systems ranking of jobs w. r. t. an overall evaluation of demands and difficulties are represented by a weighted sum of scores, which can be formally stated as follows:

$$A \succ_{v(1-n)} B \Leftrightarrow \sum w_i v_i(A) > \sum w_i v_i(B)$$

$$A \sim_{v(1-n)} B \Leftrightarrow \sum w_i v_i(A) = \sum w_i v_i(B)\,,$$

where "$\succ_{v(1-n)}$" = of more value w. r. t. an overall evaluation of all factors *1* to *n*.

"$\sim_{v(1-n)}$" = of equal value w. r. t. an overall evaluation of all factors *1* to *n*.

$v_i(A)$ = the score assigned to job *A* representing an evaluation of factor *i*.

$v_i(B)$ = the score assigned to job *B* representing an evaluation of factor *i*.

$w_i$ = weight of factor *i*.

## 3. A formal framework for evaluation of jobs

### *3.1. Jobs represented by a product set*

As mentioned above an evaluative comparison of jobs is based on a number of main criteria, which in turn are divided into a number of sub-criteria or factors. We designate an arbitrary factor by $X_i$. The specific qualitative value or level of a factor that is associated to a job is designated by $x_i(A)$, where $A$ denote an arbitrary job. Further, two jobs $A$ and $B$ can be equal or unequal w. r. t. factor $X_i$, i.e. to each factor a comparative and similarity relation are associated, which means that each factor can vary in degree. All factors are associated with at least two distinct levels. The comparative relation is designated as:

$$x_i(A) \succ_i x_i(B),$$

which is to be read as "job $A$ has a higher level than job $B$ w. r. t. factor $X_i$".

The similarity relation is designated as:

$$x_i(A) \sim_i x_i(B),$$

which is to be read as "job $A$ has an equal level as job $B$ w. r. t. factor $X_i$."

A simple example can elucidate the defined relations. Assume factor $X_i$ represents requirement of skill measured in period of training. If it is the case that the period of training for job $A$ and $B$ is $x_i(A) = 3$ years and $x_i(B) = 2$ years, respectively, then $x_i(A) \succ_i x_i(B)$, i.e. job $A$ has a higher level than $B$ w. r. t. requirement of skills. For this specific factor we can express the relation in a more simple way, as "job $A$ requires a longer period of training than job $B$".

The comparative relation and similarity relation order the jobs w. r. t. each factor. We assume that the union of both relations is a weak order on the set of jobs, i.e. the relation $\succsim$ is complete, transitive and reflexive.

Each job can thus be characterized or represented by a number of qualitative factor-levels that are associated with the job. The set of the jobs to be evaluated can be represented as a product set as follows:

$$x(A) \equiv \langle x_1(A), x_2(A), .., x_k(A), ..., x_n(A) \rangle \in X_1 \times X_2 \times .... \times X_n ,$$

where $A \in \boldsymbol{J}$ , where $\boldsymbol{J}$ is set of jobs to be evaluated.

$x_i(A) =$ a factor-level associated with job $A$ w. r. t. factor $X_i$.

$n =$ number of factors.

Assuming that all factor-levels are independently realizable means that beside actual jobs, hypothetical jobs are also contained in the product set. This means that we have a full product set, i.e. all combinations of the finite number of factor-levels are represented by the product set. Thus, out of *m* number of jobs and *n* number of factors we can receive *m* times *n* number of jobs given that all factor-levels are independently realizable. The assumption of independently realizable of factors will be discussed in section 5.

### *3.2. Inter-subjective versus subjective factors*

The distinction between inter-subjective factors and subjective factors might be important when it comes to identifying any type of subjectivity in a job evaluation process as well as to try to reduce the degree of this type of subjectivity. However, this distinction in conventional job evaluation systems seems not to be explicitly considered.

The factors can be classified as *inter-subjective factors* or *subjective factors*. The distinction is based on the type of decision method that can be applied in order to determine a factor's extension on the set of jobs. *Inter-subjective factors* are defined or specified operationally, i.e. there is an objective or empirical decision method that can be applied in order to determine the extension of the factors. An example might be the sub-factor "noise", which seems to be a relevant constituent in the main-factor "working conditions". The degrees of noise which are associated with different jobs can be determined by an objective measurement process. Thus there is little room for subjective influences on the judgments as well as for disagreements about the degrees of noise associated with jobs.

*Subjective factors* cannot by definition be associated with an operational definition or specification. The extension of subjective factors on a set of jobs is ultimately determined by the DM. An example of a typical subjective factor is "social skills". The judgment that "job A requires a higher degree of social skills than job B" is implicitly related to a specific DM. The relation to the DM can be explicitly stated "job A requires a higher degree of social skills than job B according to the decision maker C", which is of course consistent with the statement that "job B requires a higher degree of social skills than job B according to decision maker

D". But a disagreement between two decision makers cannot be solved by pointing to the result of an objective measurement process, which is possible with disagreements about extensions of inter-subjective factors. One reason for classifying "social skills" as a subjective factor is the fact that social competence is a multi-dimensional concept, which means that it is constituted by a number of sub-factors. Thus comparing two jobs w. r. t. degree of social skills means that the DM has to assess the relative influence of different sub-factors on the overall value of social skills. This means, among other things, that the DM has to decide about relative weights or importance of different sub-factors. Obviously, there is no ultimate objective measurement method that can determine the weights of sub-factors in assessing the degree of social skills. It seems that many or most of the factors used as a basis for the evaluative comparisons of jobs are typically subjective factors that cannot be associated with operational specifications.

However, the distinction between *inter-subjective factors* and *subjective factors* can be regarded as a matter of degree. At the one end of the spectrum we have factors like "noise" and at the other end we have factors like "social skills" and "responsibility". A factor between these endpoints might be "requirement of skills" defined as period of training. In contrast to measurement of "noise" there seems not to be any empirical and objective measurement method available that can determine the period of training that is required for various jobs. But on the other hand "period of training" is not defined by the vast number of sub-factors that are typical for factors such as "social skills".

### *3.3. Evaluation of jobs represented by a product structure*

The result of comparison of jobs w. r. t. an overall evaluation of the relevant factors can be represented as a value structure on the product set:

$$\left\langle x_1(A), x_2(A), .., x_n(A) \right\rangle \succ_{v(1-n)} \left\langle x_1(B), x_2(B), .., x_n(B) \right\rangle$$

or as

$$A \succ_{v(1-n)} B,$$

and

$$\left\langle x_1(A), x_2(A), .., x_k(A), .., x_n(A) \right\rangle \sim_{v(1-n)} \left\langle x_1(B), x_2(B), .., x_n(B) \right\rangle$$

or as

$$A \sim_{v(1-n)} B,$$

The relation "$A \succ_{v(1-n)} B$" is to be read as "Job $A$ is of more value than job $B$ w. r. t. an overall evaluation of the relevant factors". The relation "$A \sim_{v(1-n)} B$" is to be read as "Job $A$ is of equal value as job $B$ w. r. t. an overall evaluation of the relevant factors". The union of both relations is designated as "$A \succsim_{v(1-n)} B$", which is to be read as "Job $A$ is of at least equal value as job $B$ w. r. t. an overall evaluation of the relevant factors". The index "$v(1-n)$" means that the relations "of more value" and "of equal value" are based on an evaluation where the influence of all relevant factors is considered. The expression "overall evaluation w. r. t. all relevant factors" will from now on be termed "overall job value".

### *3.4. Overall job value as a primitive concept*

For a thorough understanding of a job evaluation process it is important to point out that the concept "overall job value" as defined above and its associated relations "of more value" and "of equal value" are to be regarded as primitive concepts in the sense that an evaluative comparison of jobs w. r. t. different factors presupposes that an overall value or a covering value, a term used by Chang (2002), is in some sense specified. If no covering value is specified, an evaluative comparison of jobs w. r. t. the relevant basis of descriptive factors seems to be a meaningless or a pointless activity. In other words, when the DM evaluates jobs w. r. t. a specific factor the evaluation is implicitly related to a covering value, which we name an overall job value. The comcept "overall job value" is the placeholder for what matters when the jobs are compared. What matters in the context of job evaluations is to give reasons for the pay setting of jobs. And to give reasons for the pay setting implies that normative principles or background norms for pay setting of jobs are at least implicitly applied (see Hare 1989 p. 141).

The importance to realizing the priority of the overall job value in order to correctly interpret a job evaluation process can be elucidated by an analogy to two evaluative comparisons of a set of alternatives w. r. t. two different overall values in terms of economic efficiency and aesthetic value, respectively. For the sake of the argument we make the somewhat strange assumption that both evaluations are based on an identical set of descriptive factors. Obviously it makes a difference when the contribution of a factor is evaluated w. r. t. economic efficiency or w. r. t. aesthetic value. An evaluation of a factor w. r. t. the overall value "economic efficiency" can of course differ in a substantial way compared to an

evaluation of the same factor w. r. t. the overall value "aesthetic value". In a similar manner it might make a difference when the contribution of the factor "educational requirement" is evaluated w. r. t. an overall value in terms of job status or w. r. t. an overall value in terms of what matters for pay setting of jobs. Thus, if the overall job value is not in some sense specified before jobs are evaluated the results of job evaluations will have an ambiguous interpretation. A more detailed analysis of the meaning and functioning of the concept "overall job value" and its relation to norms about pay setting of jobs is provided in section 6.

## 4. Evaluation of jobs by decomposing the overall job value structure

### *4.1. Decomposing the overall value structure on jobs*

When it comes to an actual evaluative comparison of jobs an important assumption is that the overall value structure is decomposable, which makes the evaluation of jobs more tractable. The assumption of a decomposable overall value structure seems to be a tacit assumption in conventional job evaluations due to the fact that the evaluative comparison of jobs are represented by additive value models as weighted sums of scores. As is well known representing the overall value structure by an additive value model presupposes that each sub-set of factors contributes to the overall job value independently of its complementary set of factors, i.e. interaction between factors regarding the contribution to the overall job value is not allowed.[3]

If *Factorial independency* holds, it is possible to determine the overall job value w. r. t each factor in a well-defined way. We define two relations. Firstly, a partial value order is defined for each factor as:

$$x_i(A) \succ_{v(i)} x_i(B) \text{ and } x_i(A) \sim_{v(i)} x_i(B),$$

which is to be read as "*job* $A$ is of more value than *job* $B$ w. r. t. $X_i$" and as "job $A$ is of equal value to $B$ w. r. t. $X_i$".

Secondly, two quaternary relations are defined for each factor as:

$$\left\langle x_i(A) x_i(B) \right\rangle \succ_{dv(i)} \left\langle (x_i(C) x_i(D) \right\rangle$$

[3] For an extensive discussion of necessary and sufficient conditions for representing value orders by additive value functions, see e.g. Keenye and Raiffa (1976) or Wakker (1989).

and

$$\left\langle x_i(A)x_i(B)\right\rangle \sim_{dv(i)} \left\langle (x_i(C)x_i(D)\right\rangle,$$

which are to be read as "the value difference between job $A$ and $B$ is greater than the value difference between job $C$ and $D$ w. r. t. factor $X_i$ ", and "the value difference between job $A$ and $B$ is equal to the value difference between job $C$ and $D$ w. r. t. factor $X_i$". These relations give rise to an order on value differences between pairs of jobs, which we term "the value difference order".

In conventional job evaluations, as *Steps to Pay Equity* described in section two, it is assumed that the value difference orders defined above are consistent with precise cardinal value structures, since the evaluations of jobs w. r. t. each factor are represented by equally spaced interval scales. It is, however, hard to believe that evaluations of a fixed number of jobs w. r. t. various factors happen to give rise to value difference orders that are consistent with equally spaced interval scales. And it seems to be the case that interval scales in terms of scores are used without any test of consistency of the qualitative value structure on jobs. Thus, the weighted sums of scores that are used as measures of the overall value of jobs are not based on well-founded "measurement processes". This means, in turn, that ranking on jobs, implied by weighted sums of scores used in conventional job evaluations, can heavily deviate from rankings on jobs implied by qualitative value structures on jobs, which are not beforehand deformed by unjustified interval scales.

### *4.2. Partial value structures induced by the overall value structure*

As discussed above, if factorial independency holds it is possible by means of the overall job value structure to define a partial value structure w. r. t. each factor which is defined by the relations, $\succsim_{v(i)}$, and, $\succsim_{dv(i)}$. The partial value structure on jobs can be summarized as:

$$\mathbf{X}_i = \left\langle X_i, \succsim_{v(i)}, \succsim_{dv(i)} \right\rangle, \text{ where } X_i = \text{a set of levels of factor } i \text{ realized by a set of jobs.}$$

However, it is important to point out that it might be the case that factorial independency holds only for the partial value order, whereas the partial value difference order might interact with other factors. A simple but realistic example can illustrate the point. We assume that the factors "heavy lifts" and "indoor temperature" are relevant for an evaluation of three jobs.

Assume further that the jobs are evaluated w. r. t. "heavy lifts" at two different indoor temperatures: $20^0C$ and $35^0C$. Assume that the value order: $x_i(A) \succ_{v(i)} x_i(B) \succ_{v(i)} x_i(C)$, is independent of the indoor temperature. At the temperature of $20^0C$ the judgment concerning the value difference order is: $\langle x_i(A)x_i(B)\rangle \sim_{dv(i)} \langle x_i(B)x_i(C)\rangle$, but at the temperature of $35^0C$ the judgment concerning the value difference order is: $\langle x_i(A)x_i(B)\rangle \succ_{dv(i)} \langle x_i(B)x_i(C)\rangle$. Thus the partial value difference order depends on indoor temperature. In other words there is an interaction between heavy lifts and indoor temperature regarding the evaluative differences between levels concerning heavy lifts.

Interactions of this type might exist for other pairs of factors, e.g. requirement of education and requirement of responsibility. For a relatively large number of factors and jobs it is of course difficult to detect all possible occurrences of interaction among factors. But interactions among factors regarded as relevant in job evaluations should not be confused with other type of dependencies that might occur. In methodological discussions about job evaluations different types of dependency conditions seem to be confused. We discuss this problem in section 5.

### *4.3. The distinction between descriptive and evaluative relations*

The important distinction between descriptive relations as "$\succ_i$" and "$\sim_i$" and evaluative relations such as "$\succ_{v(i)}$" and "$\sim_{v(i)}$" seems not to be clearly stated in job evaluations. A judgment such as "$x_i(A) \succ_i x_i(B)$" is consistent with a judgment such as "$x_i(A) \sim_{v(i)} x_i(B)$". A simple example can illustrate the distinction: Assume that $x_i(A) =$"3 years of period of training" and $x_i(B) =$"3.5 years of period of training". It is the case that both jobs could be judged to be of equal value w. r. t. skills measured in period of training, i.e. $x_i(A) \sim_{v(i)} x_i(B)$. The difference between the two jobs w. r. t. period of training is too small – ceteris paribus – there is no reason for different pay.

An important difference between the descriptive and evaluative judgment is that the last type of judgment gives a reason for the pay setting of jobs. Thus, stating that two jobs are of equal value w. r. t. skills means that the DM – ceteris paribus – finds it is reasonable that both jobs should be equally paid. Or in other words, the difference between the two jobs w. r. t.

period of training is, according to the DM, not sufficiently large to make it reasonable to pay to job *A* more than job *B*.

However, it should be stressed that for many of the basic factors in job evaluation systems it might not be meaningful to make a distinction between descriptive and evaluative relations that are associated to the factors. The reason is that many of the factors are on closer examination constituted by a set of sub-factors, i.e. the factors are aggregates of various numbers of sub-factors. For example in *Steps to Pay Equity* the factor "*Social skills*" is defined as follows:

> "Measured by: communication, co-operation, cultural understandings, empathy, service." (See Appendix).

Thus, when the DM compares jobs w. r. t. *Social skills* the DM has to assess the relative weights of the sub-factors. And when the DM assesses the weights of the sub-factors the purpose of the comparison seems to be invoked in terms of what matters. And what matters in a job evaluation is to give reasons for the pay setting of jobs, which means that assessing weights of sub-factors is to give partial reasons for pay setting of jobs. In other words, determining the extension of such a factor as *Social skills* means that an evaluative comparison of jobs w. r. t. a set of sub-factors that constitute the factor *Social skills* is carried out. Thus the extension of such multidimensional factors depends on the context. If the purpose of the comparison of jobs is changed it might be the case that the relative weights of the sub-factors are changed, which in turn means that the extension of the factor "*Social skills"* is changed.[4]

### *4.4. An interpretation of weights as applied in conventional job evaluations*

We end this section by pointing out that the functioning of numerical weights that are assigned to factors in conventional job evaluations is ambiguous. The starting point for the discussion is the observation that in conventional job evaluation systems there is no explicit definition of the notion "weight" as well as of the notion "importance". One interpretation of the intended functioning of weights is that the DM can express the opinion of the relative

[4] The discussion is based on an analysis by Griffin who asserts that "..there is no sharp separation of a natural from an evaluative component in concept such as 'accomplishment'. And purely natural descriptions is not enough to give the concept a shape, to pick out its extensions" (Griffin 1996, p. 46).

influence of various factors on the overall job value by assigning weights to the factors. But as is well known numerical weights in an additive value model are to be interpreted as scaling constants that cannot per se represent, in a meaningful way, the relative importance of factors (see Keeney and Raiffa 1976).

With the purpose of clarifying the notion "importance" we suggest a definition that is commonly applied in Multi-Criteria Decision Analysis when additive value models are specified (see Edwards and von Winterfeld, 1986 or Salo and Hämäläinen, 2001). The definition of the relative importance of factors is based on the value range between the highest and lowest ranked levels of each factor. The definitions are as:

Factor *i* is *more important* than factor *j if and only if* $\left\langle x_i^h x_i^l \right\rangle \succ_{dv(i-j)} \left\langle x_j^h x_j^l \right\rangle$

Factor *i* is *of equal importance* as factor *j if and only if* $\left\langle x_i^h x_i^l \right\rangle \sim_{dv(i-j)} \left\langle x_j^h x_j^l \right\rangle$[5],

where $x_i^h$ = highest ranked level for factor *i* and $x_i^l$ = lowest ranked level for factor *i*.

$x_j^h$ = highest ranked level for factor *j* and $x_j^l$ = lowest ranked level for factor *j*.

$\succ_{dv(i-j)}$ = the value difference between the highest and lowest ranked levels for factor *i*. is higher than the corresponding value difference between the highest and lowest ranked levels for factor *j*.

$\sim_{dv(i-j)}$ = the value difference between the highest and lowest ranked levels job for factor *i* is equal to the corresponding value difference between the highest and lowest ranked levels for factor *j*.

From the definition above it is obvious that the relation "importance" depends on the set of jobs that are evaluated in a specific situation. If the set of jobs is extended, it might imply that the value difference between the highest and lowest ranked level of one or many of the factors will change, which in turn means that the assessed importance relation has to be adjusted in a proper way. However, in conventional job evaluation systems there are no discussions about proper ways to adjust weights when the set of jobs changes or when any other relevant change occurs. This means that numerical weights as applied in conventional job evaluations have an ambiguous relation to the relative importance of factors, at least as defined above. The interpretation and intended functioning of weights in conventional job evaluations is thus highly ambiguous, something which is problematic due to the fact that assigning weights is regarded as an important stage in the job evaluation process.

[5] The definitions imply, of course, that factorial independecy holds for the overall value structure (see p. 11).

Finally, we will comment on the convention to assign precise numerical weights to the various factors. The possibility to justify such precise numerical assignment can obviously be questioned. This means that the result of a job evaluation depends on an assignment of precise weights, which cannot be justified.

## 5. Independency conditions and biased job evaluations

### *5.1. Introduction*

Methodological studies of job evaluation processes report a tendency that judgments of the DM depend on irrelevant factors which might give rise to biased evaluation of jobs not consistent with an impartial job evaluation. One such reported phenomenon is the *Halo Effect Bias*, which means that irrelevant aspects of jobs associated with positive or negative values have an influence on the evaluation of the relevant factors. One example is the evidence that DMs have a tendency to "under-value" factors and characteristics associated with typically female jobs due to the fact that female jobs have a lower status or lower wages than comparable male jobs (see Burton 1987). The dependency of job evaluation on such obviously irrelevant factors is of course important to identify and remedy. But the dependency of such influences on the job evaluation process should not be confused with other types of dependency conditions that might be important to identify. Confusing different types of dependency problems might give rise to an improper diagnosis as well as improper remedies. It seems to be the case in conventional job evaluations that high correlation among factors is confused with factorial and conceptual dependency respectively.

### *5.2. Correlation and factorial independency*

A high correlation among factors is taken as an indication that there is an interaction among factors concerning the influence on the overall job value. But a high correlation among factors is something that is to be expected. It is plausible to assume that jobs which require a high degree of responsibility also require a relatively high degree of educational levels. It might be the case that many types of responsibilities require certain educational levels. Jobs in health care seem to be typical examples. Correlation among factors because of legal or institutional constraints regarding the requirement of jobs does not imply that there are interactions among factors in terms of contribution to the overall job value. It is important to point out that the correlation among factors is related to the content of the jobs, whereas factorial independency

or dependency is an assumption about the DM's opinions about how to evaluate the contribution of various factors on the overall job value, which finally might depend on principles for pay settings that are applied in the evaluation situation. In other words, correlation among factors depends on the nature or content of the jobs, whereas factorial independency or dependency is related to principles that are applied by the DM in the evaluation situation.

In principle it might be difficult to exclude all possible interactions among factors, in particular if there is an extensive number of factors and realized levels, as is the case in job evaluations. One way to detect dependency among some factors might be by thought experiments and simple actual tests. One indication of a dependency relation between two factors is that the DM cannot make a sensible evaluation of jobs w. r. t. one factor without knowledge about jobs w. r. t. the other factors.[6] However, it should be pointed out that occurrence of dependency among factors is not necessarily based on an illegitimate evaluation process, but can instead be the result of well-justified judgments, as was illustrated above when heavy lifts were evaluated at different indoor temperatures. If two factors interact, an obvious way to maintain the possibility to decompose the overall value structure is to merge the interacting factors into one factor. This merged factor can then be partially evaluated in a well-defined way. But, of course, interactions among factors can be caused by illegitimate judgments similar to judgments explaining the *Halo Effect Bias* mentioned above. For example, it might be the case that jobs receive relative high values on most factors only due to the fact that they receive high values on an important factor, without any further justifications.

### *5.3. Correlation and conceptual dependency*

A high correlation between two factors is taken as an indication of a redundancy concerning the definition of the factors. Obviously, as explained above a high correlation between two factors does not necessarily depend on redundancy or conceptual dependency. Instead, a proper way to identify redundancies among factors is to scrutinise the definitions of the factors in order to avoid so called double counting. Double counting refers to the fact that one factor is in some sense evaluated twice, which gives it an unwarranted weight regarding the influence on the overall value. The problem with double counting can be illustrated by a simple example. Assume that one factor $X_i$ is, on closer examination, seen as determined by

[6] For a discussion about tests of factorial independency, see Keenye and Raiffa (1976) or von Winterfeld and Edwards (1986).

two basic factors $X_{1i}$ and $X_{2i}$, i.e. the main factor can be represented as: $X_i = \langle X_{1i}, X_{2i} \rangle$. Assume that another relevant factor $X_j$ is seen as determined by two sub-factors: $X_j = \langle X_{1j}, X_{2j} \rangle$. A conceptual analysis reveals that the sub-factors $X_{2i}$ and $X_{2j}$ are identical, i.e. the influence of sub-factor $X_{2i}$ is counted twice. An obvious solution in this simple example is to redefine the factors into three factors as: $X_{1i}$, $X_{2i}$ and $X_{1j}$.

Based on this brief analysis we conclude that a high correlation among factors cannot be used in order to identify factorial and conceptual dependencies among factors, i.e. occurrences of interactions and redundancies among factors. A high correlation indicates that levels of different factors are not independently realizable, which is explained by the nature or content of the jobs. The analysis also makes it evident that it is important to distinguish between factorial and conceptual dependency among factors, since different types of diagnosis are required as well as different types of remedies recommended. A remedy to an interaction between two factors might be accomplished by merging both factors, whereas a remedy to a redundancy between two factors is accomplished by redefinition of the factors by splitting them into an appropriate number of sub-factors.

## 6. The meaning and functioning of the concepts "of equal value" and "of more value"

### *6.1. Introduction*

The purpose of this section is to clarify the meaning or the functioning of the concept "overall job value" and its associated concepts "of equal value" and "of more value" as used in the context of job evaluation. Much of the debate for and against using job evaluation in order to justify pay structures consistent with e.g. *Equal Pay Acts* stems from confusions about the meaning of concepts such as "of equal value" in the context of job evaluations. The quotation below is evidence of the presence of a substantial misunderstanding concerning the meaning of the concepts "overall job value", "of equal value" and "of more value".

> "The doctrine of comparable worth rests on an assumption that each job possesses an inherent worth independent of the market forces of supply and demand."

> "These values are presumably not determined merely by someone's subjective notion of the moral worth of an activity. They are alleged to be something concrete, objective, and measurable."[7]

Postulating that each job possesses an inherent worth or value that is concrete, objective and measurable seems to be a very strange idea, at least if we interpret inherent values as intrinsic or final values. And even if jobs possess inherent values it is far from obvious in what way such inherent values give reasons for pay setting of jobs. This strange idea about inherent values is probable explained by the use of the term "value" in the context of job evaluations.

Another interpretation of the expression "inherent job value" is that jobs contribute to other important values in varying degree, i.e. jobs vary in terms of instrumental values. But interpreting the value of jobs as instrumental values seems not to be consistent with the grounds for the evaluation of jobs as stated in Equal Pay Acts. For example a factor such as bad working conditions is assumed to contribute positively to the value of jobs. It seems strange to claim that bad working conditions have a positive instrumental value. Instead, an obvious interpretation of the positive value associated with bad working conditions is that jobs involving such conditions should be compensated for by a pay increase, something that in turn can be justified by the principle of Compensating Wage Differentials.[8] Thus, values of jobs are related to reasons for pay setting of jobs, something we will elaborate further below.

### *6.2. Overall job value as an intermediary concept*

A fruitful analysis of the concept "overall job value" and its associated relations "of equal value" and "of more value" would consider these concepts as intermediaries, which can be explained as follows. Obviously, a term such as "of equal value" is evaluative. Thus it shares a common feature with other evaluative terms that have both a descriptive and an evaluative or normative meaning.[9] Another way to express this observation is to say that evaluative words such as "of equal value" serve as intermediaries between descriptive statements and evaluative statements.[10] The descriptive statements in terms of demand and difficulties associated with the jobs are the grounds for applying the concepts "of equal value", whereas

[7] The quotations above are in Arnault et al. (2001).
[8] Adam Smith, the famous 18th century British moral philospher, laid the foundations for the principle or theory of Compensating Wage Differentials (see Killingsworth 1990 for discussion).
[9] See Hare (1989) chapter 6.
[10] The discussion in this section is based on theories of intermediary concepts as developed in Lindahl and Odelstad (1996), see also Lindahl (2004).

the evaluative or normative statements are the consequences of applying the concepts "of equal value". Thus the meaning or function of the term "of equal value" in job evaluation contexts is to couple descriptive statements about relevant differences between jobs regarding demands and difficulties to evaluative or normative consequences regarding how jobs should be paid.

A simple example can illustrate the idea of analyzing the term "of equal value" as an intermediary. We assume that the DM judges that two jobs differ only w. r. t. educational requirements measured in period of training. The DM judges that *job A* requires a period of training about six months longer than *job B*, i.e.

$$x_{ed}(A) - x_{ed}(B) \approx \text{ six months.}$$

However, the DM judges that a difference of six months in period of training is not sufficient in order to claim that *job A* is of more value than *job B* w. r. t. educational requirements. Thus based on the difference observed w. r. t. educational requirement the DM judges that both jobs are of equal value, i.e.

$$A \sim_{v(ed)} B .$$

The assessment of the DM can be summarized by the following inference:

I. (1) $x_{ed}(A) - x_{ed}(B) \approx$ six months

(2) $\forall A, B \in \boldsymbol{J}$ : If - ceteris paribus - $x_i(A) - x_i(B) \approx$ six months, then: $A \sim_{v(ed)} B$

---

(3) $A \sim_{v(ed)} B$

The statement (3) can then be stated as a premise in the next inference as follows:

II: (1) $A \sim_{v(ed)} B$

(2) $\forall A, B \in \boldsymbol{J}$ : if - ceteris paribus - $A \sim_{v(ed)} B$, then should: $Wage(A) = Wage(B)$

---

(3) It should be the case that: $Wage(A) = Wage(B)$

If the second premise in both inferences is combined then an operational wage setting norm is implied as:

$$\text{If - ceteris paribus - } x_{ed}(A) - x_{ed}(B) \approx \text{ six months, then should: } Wage(A) = Wage(B)$$

The norm is named operational since the antecedent is a descriptive statement concerning differences w. r. t. educational requirements. Accepting both inferences means that the DM also has, at least implicitly, accepted the operational wage setting norm stated above. In other words before the DM concludes that both jobs are of equal value, the DM should ask if it is a reasonable wage policy to claim that a difference corresponding to six months in terms of period of training is not a reason for pay differentials between jobs. Thus, when combining both inferences the feature of the term "of equal value" as a coupling term or an intermediary becomes evident. In this case the term "of equal value" couples a descriptive difference between jobs in terms of period of training to normative consequences in terms of pay setting.

This simple example also demonstrates that a term such as "of equal value" can only be meaningfully applied if both inferences are considered. The DM with knowledge of only one of the inferences seems not to have fully grasped the functioning of the terms "of equal value" or "of more value". If the DM knows only the first inference, this means that the DM evaluates the descriptive differences between jobs without any knowledge about the purpose of the evaluation, i.e. without knowing that the result of the job evaluation is going to be used as a guideline for pay setting of jobs. And if the DM has knowledge only of the second inference, this means that the DM applies the principle "equal pay for jobs of equal value" without any knowledge of the descriptive grounds for evaluative comparisons of jobs. In other words, the DM has only a formal knowledge of the meaning of the term "of equal value". The DM knows that two jobs of equal value should be equally paid, but the DM has no idea of how to assess whether the resulting pay setting of jobs is at all reasonable, because the DM has no idea about the descriptive grounds for applying the term "of equal value".

The meaning or function of the term "of equal value" becomes evident if we use, as it seems, the equivalent formulation "considering all relevant differences between two jobs there is no reason for different pays for the two jobs". The last expression is more informative about the meaning or function of the term "of equal value" as an intermediary between grounds in terms of descriptive differences between two jobs and consequences in terms of normative statements about pay setting of the two jobs in question. This interpretation of the term "of equal value" reveals that the second premise in inference II above is true by virtue of

the meaning of the term "of equal value". The DM accepting the antecedent in the premise but denying the consequence seems to contradict him or herself, since the statement "*Job A and job B should be equally paid*" is implied by the meaning of the statement that "*Job A and job B are of equal value*". In other words, the statement "*Job A and job B should be equally paid*" can be considered as a meaning postulate for the statement "*Job A and job B are of equal value*".

However, the second premise in the first inference is not true in virtue of the meaning by the term "of equal value". The DM accepting the antecedent but denying the consequence is not contradicting his or herself. The DM accepts a different wage setting norm implying that a differences corresponding to six months of period of training is a sufficient reason for pay differential between the jobs.

If we extend the grounds to an arbitrary number of relevant factors, the following meaning postulate in terms of grounds can be stated:

(1a) If $\forall X_i \in X : x_i(A) \sim_i x_i(B)$, then $A \sim_{v(1-n)} B$.

If *job A* and *job B* are similar with respect to all relevant factors the DM is conceptually constrained to claim that *job A* and *job B* are of equal value. The second obvious meaning postulate is:

(1b) If $x_i(A) \succ_{v(i)} x_i(B)$ and $\forall X_{j \neq i} : x_j(A) \sim_j x_j(B)$, then $A \succ_{v(1-n)} B$.

If *job A* is of more value than *job B* w. r. t. factor *i* then - ceteris paribus - the DM is conceptually constrained to claim that *job A* is of more value than *job B*.

In terms of normative consequences the following meaning postulates can be stated:

(2a) If $A \sim_{v(1-n)} B$, then should: $Wage(A) = Wage(B)$.

(2b) If $A \succ_{v(1-n)} B$, then should: $Wage(A) > Wage(B)$.

The statement (1a-b) and (2a-b) can thus be considered as partial definitions of the concepts "of equal value" and "of more value". The statements represent the minimum conceptual

constraints for the application of the concepts "of equal value" and "of more value" in the context of job evaluations.

It is informative to compare these meaning postulates with a substantial normative statement as follows:

(3a) If - ceteris paribus - $x_i(A) \succ_{v(i)} x_i(B)$ and $x_j(B) \succ_{v(j)} x_j(A)$, then $A \succ_{v(1-n)} B$,

i.e. if *job A* is of more value than *job B* w. r. t. factor *i* and *job B* is of more value than *job A* w. r. t. factor *j*, but with respect to the other factors the two jobs are exactly similar, then *job A* is of more value overall than *job B*. This is obviously not implied by the meaning of the concept "of more value". The statement that, all things being considered, *job A* is of more value than *job B* is a substantial normative assessment to the effect that the value difference w. r. t. factor *i* is more important than the value difference w. r. t. factor *j*. This means that the DM, who accepts the antecedent in statement (3a), but denies the consequence, does not contradict him or herself. Such a DM expresses a different normative opinion about pay setting of jobs.

Next statement illustrates an important difference between similarities w. r. t. descriptive aspects and similarities w. r. t. evaluative aspects of jobs.

(3b) If - ceteris paribus - $x_i(A) \sim_{v(i)} x_i(B)$ and $x_j(A) \sim_{v(j)} x_j(B)$, then $A \sim_{v(1-n)} B$.

The statement seems to be identical to statement (1a) above, but there is an important difference. In statement (3b) it is not claimed that *job A* and *job B* are descriptively similar w. r. t. factor *i* and *j*. What is stated is that - ceteris paribus - there is no reason to value *job A* and *job B* differently w. r. t. factor *i* and factor *j*, respectively. But in conjunction an evaluation w. r. t. factor *i* and *j* might support the judgment that *job A* is of more value than *job B*, without any conceptual contradictions being involved. It might be the case that w. r. t. each factor - ceteris paribus - the descriptive differences between *job A* and *job B* are too small to support the judgment that *job A* is of more value than *job B*. But in conjunction the differences overall have passed a "threshold", which can justify that *job A* is of more value than *job B*. It seems strange to exclude such an interaction across factors due to conceptual constraints. Thus the following statement is not a contradiction:

(3c) If - ceteris paribus - $x_i(A) \sim_{v(i)} x_i(B)$ and $x_j(A) \sim_{v(j)} x_j(B)$, then: $A \succ_{v(1-n)} B$.

The statements (3b) and (3c) can be explained by the fact that two different pay setting principles are applied.

We conclude that by using the theories of coupling terms we can establish a reasonable explanation of the meaning and function of the key concepts "overall job value" and its associated relations "of equal value" and "of more value" compared to postulating the existence of an inherent job value. Thus the function of the term "of equal value" in the principle "Equal Pay for Jobs of Equal Value" is to couple the overall judgments of jobs w. r. t. demands and difficulties to the normative recommendation that the jobs should receive equal pay. Further, the analysis in terms of coupling terms also makes it explicit that evaluation of jobs presupposes or invokes normative principles about pay setting of jobs. In other words, this conceptual analysis shows that normative principles about pay setting have in some sense a priority when jobs are evaluated.

### *6.3. Using the concept "overall job value" in a narrow or in a wide sense*

The concept "overall job value" and its associated relations "of equal value" and "of more value" can and seem to be used in both a narrow and wide sense, which gives rise to confusions when the results of job evaluations are to be implemented. Using the concept "of equal value" in a wide sense means that there is no other reason for the pay setting of the jobs, i.e. all relevant differences between the jobs and other facts are considered in the assessment that *job A* and *job B* are of equal value. Using the concept "of equal value" in a narrow sense means that there can be other reasons for the pay settings besides what is considered in a job evaluation.

The difference in using the concept in a wide or narrow sense can be illustrated by a disagreement between two parties - an employer and a representative for the employees. Both parties can agree that two jobs are of equal value w. r. t. a set of factors *i* to *n*, i.e. they agree that:

$$A \sim_{v(1-n)} B.$$

But they can of course disagree about the consequences of the statement. An employer, who uses the concept in a narrow sense, might claim that *job A* should be given a higher pay than

*job B*, because there are other relevant differences between the two jobs, which are not included into the grounds for applying the concept "of equal value".

If both parties use the concept "of equal value" in a wide sense this means they agree about the consequences of applying the concept "of equal value", i.e. they accept the pay setting principle stating that:

$$\forall A, B \in \mathrm{J}: \text{If } A \sim_{v(1-n)} B \text{, then should: } Wage(A) = Wage(B).$$

But, of course, even if both parties accept this principle, they can come to different conclusions concerning the value of the two jobs. This can happen, even if both parties start from a set of identical factors, because both parties can apply different background norms when jobs are evaluated w .r. t. various factors as well as when weights are assigned to the factors.

### *6.4. The definitions of factors and pay setting norms*

As follows from the discussion above, the evaluation of jobs presupposes norms – implicit or explicit – that are applied in various stages of an evaluation process. But it is important to realize that normative principles might also have an important impact on the way criteria or factors as stated in e.g. Equal Pay Acts are defined. This means that norms, besides influencing the evaluation process, determine the basis for evaluation of jobs in terms of defined factors. As we demonstrate below, starting from different norms might give rise to different definitions of suggested factors.

Already the basic term "jobs" seems to be open to different explications depending on a choice of principle for wage setting, which might, in turn, depend on different interests involved in a wage setting process.[11] From an employer's point of view it is the contribution to the production value that seems to be a relevant starting point or basic aspect when it comes to evaluating jobs, the purpose of which is to give reasons for pay setting. But from the employees' point of view it is the effort required for doing the job that seems to be the relevant starting point or basic aspect. These two different points of views about what is the essential basis for job evaluations have implications for definitions of the factors that are stated in e.g. the Equal Pays Acts. We illustrate this by suggesting two different definitions of the evaluation of the factor: *requirement of skills*. A basic question seems to be: How should

[11] A similar discussion is in Killingsworth (1990) chapter 2.

*requirement of skills* be defined? Depending on interests of the parties the question might have different reasonable answers or in other words the factors are essentially contested.[12]

From the employer's point of view a reasonable definition of the value of *requirement of skills* is:

$$A \succ_{v(skills)} B$$

*if and only if*

> "Production value losses that occur if skills required in *job A* are not available *are greater than* the corresponding losses that occur if skills required in *job B* are not available."

In other words, the relations "of equal value" or "of more value" w. r. t. *requirement of skills* are defined in terms of production value losses, which in turn can be defined in various ways depending on context. For a typical firm producing goods value losses might ultimately be defined in terms of expected losses of profit. But for a hospital value losses might ultimately be defined in terms of losses of quality of life according to patients or to any other agents. Thus, according to the definition, jobs associated with relative high value losses should be given relatively high pays. The rationale for recommending higher pays for jobs associated with higher production value losses might be that a higher pay decreases the risk for occurrences of heavy production value losses. Such a rationale for evaluation and pay setting of jobs is consistent with efficiency wage theories, which play an important role in labor economics in order analyze the wage setting processes on labor markets.[13]

From the employees' point of view a reasonable definition of the value of *requirement of skills* might be:

$$A \succ_{v(skills)} B$$

*if and only if*

> "Costs of acquiring skills required in job *A* *are higher than* costs of acquiring skills required in job *B*."

[12] The theory of essentially contested concepts is well known from Gallie (1956), see also Chang (2002), p.169.

This definition is consistent with the principle of compensating wage differentials. The definition also gives a reason for the fact that in conventional job evaluations *requirement of skills* is usually estimated in terms of the period of training, which seems to be a good estimate of costs for acquiring the skills. Thus, besides practical reasons in terms of easiness to acquire information about *requirement of skills,* there is also a principle of normative reason for evaluating *requirement of skills* in terms of period of training.[14] In other words, this definition of requirement of skills seems not to be value-neutral in the context of job evaluation.

Discussion about what norms for wage settings are or should be applied when jobs are evaluated, the purpose of which is to reveal indications of a gender biased pay setting is beyond the scope of this study. But nevertheless the formal analysis in this section reveals an important and fundamental feature of job evaluation and associated key concepts as "of equal value" and "of more value", namely that evaluation of jobs necessarily invokes principles for pay settings that have a priority in the evaluation process.

## 7. "Equal Pay for Jobs of Equal Value" and imprecise comparisons of jobs

### *7.1. Introduction*

In this final section we discuss the problem of an application of the principle "Equal Pay for Jobs of Equal Value" considering the possibility that jobs can only be imprecisely compared. We start from the observation that it is common in job evaluations to determine the rankings of jobs by means of weighted sum of scores, where the scores represent partial evaluation of each factor. If the numerical model intends to represent the relations "of more value" and "of equal value" it implies that the union of both relations gives rise to a weak order on jobs, which among other things implies that:

For all jobs $A, B, C \in J$: If $A \sim_{v(1-n)} B$, $B \sim_{v(1-n)} C$ then $A \sim_{v(1-n)} C$.

But is it possible for the DM to justify such precise comparisons? This can be questioned since in the context of job evaluations the DM's final judgment expressed by "of more value" or "of equal value" is based on overall evaluation of a large number of factors with ambiguous definitions and imprecise relative importance. Even in a much simpler comparison

[13] See Akerlof and Yellen (1986) and Weiss (1991) for surveys of Efficiency Wage Theories.
[14] For extensive discussion about relations between pay setting norms and definition of factors, see Soltan (1987).

regarding "subjective loudness" it is easy to find three sounds *a, b, c* such that a person judges *a* and *b* as well as *b* and *c* to be equally loud, while the person judges *a* to be louder than *c*.[15] The observed non-transitivity can be explained by the fact that a person cannot distinguish between too small differences in objective loudness. The difference has to pass a threshold, the so called "just noticeable difference", before a subject can perceive a difference in subjective loudness. In analogy to judgements of "subjective loudness" it seems realistic to assume that for some jobs *A, B* and *C,* the DM finds no reason for deciding that *A* and *B* as well as *B* and *C* are of *unequal value*, while the same DM finds reasons for deciding that *A* is *of more value* than *C.* The non-transitivity can be explained by the fact that in the context of job evaluation no precise but only rough comparisons can be justified. For that reason a more adequate name of the relation "of equal value" might be "of roughly equal value".[16]

*7.2. Evaluation of jobs giving rise to semi-orders*

The relation "of roughly equal value" can be defined as:

$$A \approx_{v(1-n)} B \text{ } \textit{iff} \text{ not } ( A \succ_{v(1-n)} B ) \text{ and not } ( B \prec_{v(1-n)} A ).$$

Given this definition the principle "Equal Pay for Jobs of Equal Value" might be adjusted to "Equal Pay for Jobs of Roughly Equal Value", i.e.

$$\text{If } A \approx_{v(1-n)} B \text{ then should: } Wage(A) = Wage(B).$$

But due to non-transitivity of the relation "of roughly equal job value" an application of the principle of Equal Pay can give rise to counter-intuitive wage equalization among the set of evaluated jobs, which we illustrate by the following example. Assume that the result of an evaluation of three jobs is as follows:

$$A \approx_{v(1-n)} B, \; B \approx_{v(1-n)} C \text{ and } A \succ_{v(1-n)} C.$$

To make this result consistent with the principle of Equal Pay means that all three jobs have to be given the same pay. Further, it might be the case that all jobs or at least a large part of the

[15] The example is discussed in Roberts (1979, p. 248).
[16] Parfit (1984, p. 431) discusses comparisons that can be "rough" and defines a relation he name "roughly equally good", which is in contrast to "equally good" a non-transitive relation.

jobs at a workplace are partially ordered by the relation "of more job value" in a descending order as follows:

$$A_i \approx_{v(1-n)} A_{i+1},\ A_{i+1} \approx_{v(1-n)} A_{i+2} \text{ and } A_i \succ_{v(1-n)} A_{i+2},$$

where $i = 1$ to $n$.

A consistent application of the principle of Equal Pay implies that all $n$ jobs at the workplace should be equally paid, which obviously can be unacceptable pay equalization.

However, the problem due to non-transitivity of the relation "of roughly equal value" can be solved if the relation "of more value" is a semi-order.[17] In that case it is well known that a weak order can be associated by a general definition in terms of the semi-order.[18] Based on the specific descending order illustrated above it is obvious that a weak order can be defined as:

$$A_i \succ_{v(1-n)} A_{i+1} \textit{ iff } A_i \approx_{v(1-n)} A_{i+1},\ A_{i+1} \approx_{v(1-n)} A_{i+2} \text{ and } A_i \succ_{v(1-n)} A_{i+2}.$$

Obviously, by using the value information contained in an associated weak order we can avoid such counter-intuitive wage equalization due to intransitivity of the relation "of equal value".

But there are two problems that have to be pointed out when the application of the principle of Equal Pay is based an associated weak order. Firstly, the associated weak order is unstable when the set of jobs to be evaluated is changed, which we illustrate by a simple example. Assume that two jobs have been evaluated as:

$A \approx_{v(1-n)} B$, which trivially implies the associated weak order: $A \sim_{v(1-n)} B$.

Assume that a third job $C$ is included and the result of the evaluation is as follows:

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[17] A binary relation R is a semi-order iff for all $a,b,c,d \in A$

*i*) $not(aRa)$ *ii*) $aRb\ \&\ cRd \Rightarrow (aRd\ or\ cRb)$ *iii*) $aRb\ \&\ bRc \Rightarrow (aRd\ or\ dRc)$ (see Roberts 1979 chapter 6).

[18] Given a semiorder R an associated weak order W is defined as:

aWb iff $(\forall c \in A)\left[(cRa \Rightarrow cRb)\ \&\ (bRc \Rightarrow aRc)\right]$ (see Roberts 1979 chapter 6).

$$A \approx_{v(1-n)} B,\ B \approx_{v(1-n)} C \text{ and } A \succ_{v(1-n)} C,$$

which implies by the associated weak order that:

$$A \succ_{v(1-n)} B.$$

Thus the application of the principle of Equal Pay implies in the first case that the jobs $A$ and $B$ should be equally paid, whereas in the second case the two jobs should be paid differently. Despite the simplicity of the example it has strong implications for the outcome of legal proceedings concerning wage discrimination by gender. A decision in the court that two jobs are of equal value can depend on whether a third job is included in the evaluation.

Secondly, it is important to point out that the associated weak order is based on a formal definition, which works if the partial order established by the relation "of more job value" is a semi-order. This means that the conclusion based on the associated weak order that job $A$ is of more job value than $B$ is not finally based on an overall evaluation of the differences between the two jobs. To see the problem, we can compare the use of a weak order associated with the relation "a higher degree of subjective loudness", which is a semi-order. In this case the associated weak order can be used to uncover an underlying structure on objective loudness. But concerning job evaluation there is no underlying objective and precise value structure that can be uncovered by a well defined associated weak order. An associated weak order can instead give rise to in some sense unjustified conclusions about the relative values of jobs, which we illustrate by the example used above:

$$A \approx_{v(1-n)} B,\ B \approx_{v(1-n)} C \text{ and } A \succ_{v(1-n)} C.$$

Starting from this evaluation result we assume that the DM succeeds in constructing more precise evaluation criteria or getting more accurate information. Beside the associated weak order,

$$A \succ_{v(1-n)} B \succ_{v(1-n)} C,$$

the outcome of the re-evaluation can be:

$$B \succ_{v(1-n)} A \succ_{v(1-n)} C \text{ or } A \succ_{v(1-n)} C \succ_{v(1-n)} B.$$

It seems to be an open question which of these three rankings will be established if the DM can re-evaluate the jobs by using more precise evaluation criteria. However, if the DM has not got access to more precise evaluation criteria, then an associated weak order might be a justified solution in order to avoid the outcome of counter-intuitive wage equalization as illustrated above.

*7.3. Evaluation of jobs giving rise to interval orders*

But even if the application of an associated weak order is defendable it presupposes that the relation "of more value" is a semi-order. However, there are reasons to believe that the relation "of more value" is not consistent with a semi-order. For example the following partial order is not consistent with a semi-order:

$$A \succ_{v(1-n)} B \succ_{v(1-n)} C,$$

and

$$A \approx_{v(1-n)} D, B \approx_{v(1-n)} D \text{ and } C \approx_{v(1-n)} D.$$

The first three jobs are ordered by the relation "of more value", whereas job $D$ is judged to be of roughly equal value compared to the three first jobs. The partial order in the example is consistent with an interval order, which is a generalization of a semi-order.[19]

But is there any reason to believe that the relation "of more value" gives rise to such a partial order? The following example makes is it plausible to assume that an application of the relation "of more value" can give rise to such a partial order. We assume that three jobs, $A, B$ and $C$, belong to the same occupational category, e.g. medical technicians employed at a hospital, and a fourth job, $D$, corresponds to a hospital nurse. We assume that the three jobs in the first occupational category are almost similar w. r. t. all factors except w. r. t. the factor *"requirement of skills"*. In other words, we assume that the only relevant difference between the jobs is based on *requirement of skills*. Within an occupational category an operational specification of the relation "*degree of requirement of skills*" can be suggested as follows:

[19] An interval order R is defined as: for all $a, b, c, d \in A$: *i)* $not(aRa)$ *ii)* $(aRb \ \& \ cRd) \Rightarrow (aRd \ or \ cRd)$, see Roberts (1979) chapter 6.

"Job $A$ requires more skills than job $B$"

*if and only if*

"skills needed for carrying out job $A$ suffice in order to carry out job $B$, but not vice versa."

We assume that the three jobs can be represented by skill profiles as:

$$x_{sk}(A) = \langle sk_3, sk_2, sk_1 \rangle,\ x_{sk}(B) = \langle none, sk_2, sk_1 \rangle \text{ and } x_{sk}(C) = \langle none, none, sk_1 \rangle,$$

where $sk_3$, $sk_2$ and $sk_1$ designate three skill levels. The skill levels represent in some sense a carrier ladder. Thus, when the three jobs are compared w. r. t. *requirement of skills* the DM can apply the operational specification giving rise to the order:

$$x_{sk}(A) \succ_{sk} x_{sk}(B) \succ_{sk} x_{sk}(C).$$

Note that the difference between the skill levels can be very small e.g. measured in terms of the period of training. The third and second skill level might for different reason just correspond to one day of training, which nevertheless is necessary for holding the jobs. Thus, the DM has access to a relatively precise evaluation criterion w. r. t. *requirement of skills* when jobs are compared belonging to the same occupational category. Despite small differences e.g. in period of training the DM can give good reasons for valuing the three jobs differently and then give reasons for pay differentials between the three jobs by pointing at an objective or inter-subjective descriptive differences between the jobs. In the example it might be the case the DM finds the differences between jobs w. r. t. *requirement of skills* not sufficient per se to give reasons for different overall jobs values and thus for different pay setting of the jobs. But in interaction with other descriptive differences between the jobs based on similar operational specifications a justified overall ranking of the jobs can be established.

But when two jobs from different occupational categories are to be compared w. r. t. *requirement of skills* the operational specification stated above could not be applied. For example, when medical technicians and nurses are to be compared w. r. t. *requirements of skills* the operational specification stated above does not work. Or in other words, the two jobs

are incomparable w. r. t. the operational specification stated above, i.e. both jobs are neither of unequal nor of equal value. Thus, when jobs from different occupational categories are to be compared the DM has to apply other operational specifications of requirement of skills as e. g. estimations of period of training, which are used in conventional job evaluation systems as *Steps to Pay Equity* described in section 2.

Usually estimations of period of training are given as numerical intervals, the length of which represents the uncertainty about the true period of training. If the estimated numerical intervals of two jobs overlap then the DM has no reason to evaluate the jobs differently w. r. t. *requirement of skills*. In conventional job evaluations it is also common to claim that two jobs are of equal value if the difference in period of training is too small e.g. a difference six months.

Possible consequences of a change of operational specifications when different types of jobs are to be compared can be illustrated by the example above. When job $D$ is compared with each of the jobs $A, B$ and $C$ w. r. t. requirement of skills estimated in period of training the DM might find that period of training for job $D$ overlaps with the period of training for each of the three jobs belonging to the other occupational category, which implies:

$$A \approx_{v(sk)} D, B \approx_{v(sk)} D \text{ and } C \approx_{v(sk)} D.$$

But when jobs within the same occupational category are evaluated w. r. t. requirements of skills the DM can shift to a more precise operational specification as defined above, which makes the DM to conclude that:

$$x_{sk}(A) \succ_{v(sk)} x_{sk}(B) \succ_{v(sk)} x_{sk}(C).$$

A plausible generalization based on the example is that when evaluating jobs belonging to the same occupational category it is possible to make more precise comparisons of the jobs w. r. t. various factors than that which is possible when jobs compared belonging to different occupational categories. The precision of evaluative criteria as a function of what types of jobs that are compared can thus give rise to partial orders not consistent with either weak orders or semi-orders. Further, in the simple example an interval order is established, but if we increase the number of jobs that belong to different occupational categories a partial order

might arise that is not even consistent with interval orders.[20] Obviously, the principle of "Equal Pay for Jobs of Equal Value" cannot be applied in sensible way if such partial orders are established.

However, in conventional job evaluations, where the overall job value is represented in terms of weighted sum of scores, such problematic orders will not arise. But the possibility of using different operational specifications related to different discrimination capacity means that the results of job evaluation are in some sense unstable. The results of job evaluations might depend on the types of jobs that are to be evaluated as well as other arbitrary decisions. We illustrate this concern about instability by using the example above.

Firstly, we assume that an evaluation occurs for jobs within an occupational category as: $\{A, B, C\}$. The result of the evaluation is: $A \succ_{v(1-n)} B \succ_{v(1-n)} C$ and the recommended pay setting is:

$$Wage(A) > Wage(B) > Wage(C).$$

Secondly, we assume that an evaluation occurs for jobs belonging to different occupational categories as: $\{A, B, C, D\}$. The result of the evaluation is: $A \approx_{v(1-n)} B \approx_{v(1-n)} C \approx_{v(1-n)} D$. The recommended pay setting is:

$$Wage(A) = Wage(B) = Wage(C) = Wage(D).$$

The difference between the results of the two evaluations is explained by the fact that two different operational specifications with different discrimination capacities are applied.

Thirdly, we assume that the evaluation occurs in two steps. First, the DM starts to evaluate the three jobs belonging to the same occupational category, which implies: $A \succ_{v(1-n)} B \succ_{v(1-n)} C$. Next, the DM can decide to compare job $D$ belonging to the other occupational category with either job $A$ or $C$, which implies that: $A \approx_{v(sk)} D$ or $C \approx_{v(sk)} D$. If the DM assumes that the relations "of roughly equal value" and "of more value" form a transitive relation, then based on the pair wise comparisons the recommended pay settings are as:

[20] See Danielsson (1998) for a similar analysis of some basic problems about numerical representations of value-orderings.

$$Wage(D) = Wage(A) > Wage(B) > Wage(C)$$

or

$$Wage(A) > Wage(B) > Wage(C) = Wage(D).$$

The example gives rise to four different outcomes of a job evaluation. The four different outcomes depend on in some sense arbitrary decisions as to what type of operational specification that can and should be applied and in what order the evaluation should be carried out. The impact of such arbitrary decisions on the evaluation process will of course increase when a large number of jobs belonging to different occupational categories are evaluated w. r. t. a relative large number of various factors or criteria.

We conclude the section by claiming that these type of problematic value orders discussed above and subsequent instability in the job evaluation process will not be observed in conventional procedures as *Steps to Pay Equity*. However, it is beyond the scope of this study to evaluate the consequences of these findings for a consistent and rational application of the principle "Equal Pay for Jobs of Equal Value".

## 8. Conclusions

In this paper we have discussed and analyzed a number of assumptions and conceptual issues that arise in applications of conventional job evaluations, which are used in order to implement the principle "Equal Pay for Jobs of Equal Value" according to Equal Pay Acts.

Firstly, in conventional job evaluations rankings of jobs w. r. t. an overall evaluation of a set of relevant factors are represented by an additive value function in terms of weighted sums of scores. An application of additive value functions implies that the overall job value structure is decomposable. i.e. factorial independency has to hold. We demonstrate that it is important to make a distinction between factorial independency regarding partial value orders and partial value difference orders defined for each of the factors. Factorial independency might hold for partial value orders but not necessarily for partial value difference orders.

Secondly, the distinction between subjective and objective factors as well as between descriptive and evaluative criteria are not explicitly stated in conventional job evaluations. This means that discussions about the impact of subjectivity in the job evaluation process suffer from ambiguities.

Thirdly, diagnosis and remedies of methodological defects suffer from confusions between different types of dependency relations among factors. We demonstrate that occurrences of

factorial and conceptual dependencies, respectively, require different types of diagnosis as well as different types of remedies. These two dependency relations have to be distinguished from a correlation among factors, which depends on the content or nature of the jobs.

Fourthly, we argue that the meaning or functioning of concepts as "of equal value" and "of more value" suffers from obvious misinterpretations in conventional job evaluations. We suggest an alternative interpretation based on theories of intermediary concepts as follows. The function of the concept as "of equal value" is to couple grounds in terms of demand and difficulties associated with the jobs to consequences in terms of norms for pay setting of jobs. The conceptual analysis also makes it clear that the application of concept as "of equal value" presupposes or invokes background norms or principles for pay setting of jobs.

Fifthly, in conventional job evaluations it is assumed that the ranking of jobs is consistent with a weak order, which is implied by representing the overall job values by weighted sums of scores. We argue that the value orders on jobs have more complex structures not consistent with weak orders. By using simplified examples we demonstrate that it is plausible to assume that the value orders on jobs are consistent with semi-orders or interval orders. Both types of orders might be difficult to reconcile with an application of the principle "Equal Pay for Jobs of Equal Value". Further, we also demonstrate that the complex value structures give rise to instability of the results of job evaluation, which thus ultimately seems to depend on a number of arbitrary decisions.

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**Appendix:** The factor plan as defined in *Steps to Pay Equity*

The basic version of *Steps to Pay Equity* is based upon MAIN AREAS, factors and aspects

**SKILL**

Factor 1. Education/experience measured by: *number of years of education, occupational experience, further education*
Factor 2. Problem solving measured by: *type of problem, creativity, independence, decision-making, development, versatility*
Factor 3. Social skills measured by: *communication, co-operation, contacts, cultural empathy, service*

**RESPONSIBILITY**

Factor 4. Responsibility for material resources and information measured by: *financial value, what the responsibility entails, independence, sequences*
Factor 5. Responsibility for people measured by: *what the responsibility entails, independence, consequences*
Factor 6. Responsibility for planning, development, results, management measured by: *the focus and scope of the responsibility, independence,*

WORKING CONDITIONS

Factor 7. Physical conditions measured by: *physical strain, strain on the senses, unpleasant physical conditions, risk for personal injury or illness*
Factor 8. Mental conditions measured by: *concentration, monotony, availability, trying relationships, stress*