Responsiveness of Swedish housing prices to the 2018 amortization requirement.
An investigation using a structural Vector autoregressive model to estimate the impact of macro prudential regulation on the Swedish housing market.

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Abstract
This thesis analyzed and estimated the impact of the March 1, 2018 loan to income amortization requirement on residential real estate prices in Sweden. A four variables vector autoregressive model (VAR) was used to study the relationships between residential real estate prices, GDP, real mortgage rate and consumer price index over a time period from 2005 to 2017. First, a structural vector autoregressive (SVAR) model was used to test how a structural innovation in the error term for real mortgage rate affected residential real estate prices. Secondly, an unconditional forecast from our reduced VAR was produced to estimate post 2017 price growth of the Swedish housing market. The impulse response function results stand in contradiction to economic intuition i.e. the price puzzle problem. The unconditional forecast indicates that the housing market will enter a period with slower price growth post 2017, which are in line with previous research. This thesis vector autoregressive model can give meaningful results with regard to trend forecasts but with regard to precise statements as anticipating drastic price depreciation, it falls short. We recommend the use of reduced VAR forecasting with regard to the Swedish housing market.

Key words: Macroeconomics, inflation rate, interest rates, house prices, econometrics, time series analysis, SVAR analysis, forecasting.
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1 Introduction
Swedish house prices and subsequently household debt levels have seen an almost uninterrupted growth trend since the early 1990s. The value of the Swedish housing market has had an average annual compounded growth rate of 7.84 per cent during 2005-2017\(^1\). During this same period, inflation adjusted disposable income increased by 2.52 percent year over year.

During the years of 2007-2008 (the financial crisis), it was apparent that the past booming residential market and high growth rates in the quantity of mortgages issued, resulted in greater defaults and a large debt risk which resulted in many negative impacts to the broader economic development of Sweden. This occurrence put pressure on politicians to implement more macro prudential policies to reduce and further mitigate the growth in household debt specifically associated with increasing house prices, and more so mortgages to fulfil these purchases. One such policy method was the implementation of mortgage loan regulation. Specifically, Sweden’s financial supervisory authority (FI) introduced a Loan to value (LTV) ceiling in 2010 and subsequently in 2016, implemented LTV amortization intervals.

In 2014, the national bank of Sweden produced a report that investigated the effects of two different amortization regulation on the housing market, GDP, and consumption, using a dynamic stochastic general equilibrium model. This model predicted a negative short term effect on the Swedish housing market ranging between 5-12 per cent, depending on the type of regulation implemented (Sveriges riksbank. 2014). It concluded the long term effects were negligible.

Later, Sweden's financial supervisory authority produced a report *Consequences of a stricter amortisation requirement* (finansinspektionen, 2017) where they calculated elasticities to model the impact of the 2018 amortization type requirement on household debt and house prices. This report concluded that the amortization requirement would decrease the growth rate of house prices by 1.5 percent nationwide, post implementation.

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\(^1\) Calculations made by the authors of the thesis.
This thesis will conduct a similar analysis as the aforementioned report but will apply a vector autoregressive (VAR) model on the same data set. The VAR model was introduced by C.A. Sims in 1980 as a response to macroeconomic modeling at the time. Today, the VAR model is widely acknowledged to be a good methodology to forecast and model policy implications on macroeconomic variables. Sims promoted the VAR models as providing a theory free method to estimate economic relationships, therefore being an alternative to the "incredible identification restrictions" in structural models (Sims, 1980).

This thesis will investigate the impact of FI’s latest macro prudential regulation - the March 1, 2018 amortization regulation with regard to limits in LTV. And its reflective short and long term effect on house prices through a VAR model application.

1.1 Background
The mortgage loan is often the biggest debt position on household balance sheets (Finansinspektionen, Dnr 2017-046). Financial regulators talk about two ratios – loan to value (LTV) and loan to income (LTI). Finansinspektionen implemented a mortgage loan ceiling in 2010, which meant that one could not borrow more than 85 percent of the value of a property.

On June 1, 2016 further amortization regulation were issued to restrict LTV ratios. For example, a household with LTV ratio in the interval of 70-85 percent could amortize 2 percent yearly. A household with a LTV ratio in the interval 50-70 percent could amortize 1 percent yearly (Finansinspektionen, Dnr 2017-118).

This changed on March 1, 2018. Prior to this, regulation was only focused on LTV ratios. However, the new regulation built on the past and specified the amount a household needed to amortize with respect to LTI ratio. This regulation will only affected new loans and refinancing of existing loans.

These regulations were implemented with the thought of improving the macroeconomic robustness of Swedish households and reducing overall household debt. It was hoped these result would make the Swedish economy more capable to withstand different negative economic shocks and stresses, thereby reducing real economic impact and smoothing out the tops and bottoms in the business cycle (Finansinspektionen, 2017).
With the aforementioned new requirement the financial authorities also introduced a new dimension which further disincentives extensive borrowing. This was through setting limits defined both with income as well as debt to asset price ratios.

Any positive or negative ripple effect on the housing market can have an effect on the debt/house price ratio which (as seen in Table 1.1) can change the individual household’s amortization obligations.

Subsequently the possible ripple effects theorized by konjunkturinstitutet, 2015 suggested the use of VAR for estimation, as this would encapsulate and highlight the broader relationship between various macroeconomic variables.

### 1.2 Purpose
The purpose of this thesis is to investigate the estimated effects of the implementation of the March 1, 2018 amortization regulation with regard to the LTI ratio on house prices. The results of this thesis will be compared and contrasted to the Swedish Riksbank and FI reports respectively.

### 1.3 Question formulation
How does house prices respond to the new amortization regulation with regard to variables such as mortgage loan rate, GDP, and inflation, within a structural vector autoregressive model?

How well can a reduced VAR forecast the development of the housing market and how well does the trend forecast compare with the impulse response?

### Table 1.1 Amortization structure source: (Finansinspektionen, 2017)

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th>AMORTIZATION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DEBT/INCOME</strong></td>
<td></td>
</tr>
<tr>
<td>ABOVE OR EQUAL TO 450 %</td>
<td>1 %</td>
</tr>
<tr>
<td><strong>DEBT/HOUSE PRICE</strong></td>
<td></td>
</tr>
<tr>
<td>0-50%</td>
<td>0</td>
</tr>
<tr>
<td>50-70%</td>
<td>0</td>
</tr>
<tr>
<td>70-85% AND ABOVE</td>
<td>0</td>
</tr>
<tr>
<td><strong>TOTAL AMORTIZATION</strong></td>
<td>1 %</td>
</tr>
</tbody>
</table>

With the aforementioned new requirement the financial authorities also introduced a new dimension which further disincentives extensive borrowing. This was through setting limits defined both with income as well as debt to asset price ratios.
1.4 Method
This thesis will employ a structural vector autoregressive model in order to test the implications of the new amortization regulation on our chosen economic variables mentioned previously. The objective is to isolate one variable and test how a structural innovation in the error term will affect our variables through a dynamic iterative process. One of the main underlying goals of this thesis, is to investigate how the housing market responds to structural innovation in the lending rate. For this thesis the shock in the error term will represent the increased cost of lending through the implementation of increased mortgage loan amortization. It should be noted that even though loan service cost and amortization are two different variables, the mortgage rate will serve as a carrier variable to introduce the amortization effect in this thesis’s model. As well as impulse response, we will produce an unconditional forecast to see how the reduced VAR model would forecast the short term price development on the Swedish housing market.

This thesis will use a vector autoregressive (VAR) model for analysis. More specifically, a structural vector autoregressive (SVAR) model will be used to investigate and produce an impulse response function. The unconditional forecast will be done by the estimated reduced VAR model. The SVAR is a form of the VAR model that is widely used as a tool to forecast macroeconomic development through a multivariate approach, treating all variables under consideration as endogenous. The variables are regressed with respect to its own lag and the lag of all the other variables investigated. The production of the VAR model is similar to the production of the autoregressive (AR), or autoregressive integrated moving average (ARIMA), processes. This thesis will use EViews ® 9 as the statistical software to create the model. Absolute percentage error (MAE) among others, will be used to compare the estimated model with data not used in the estimation process. This thesis will use Swedish monthly and quarterly data from 2005-2017 and test this models validity on the eight quarters ranging between 2016-2017.

1.5 Data sources
This thesis will use Swedish quarterly and monthly data during the period 2005 - 2017. This data is taken from the Statistics Sweden and KTH-Valueguard. Reference to these data sources are in appendix A. The measurement frequency of the three of four chosen variables are taken on a monthly basis. The monthly data will be transformed to quarterly data points by taking the arithmetic average. The four chosen variables are:
**Inflation** - The data for inflation is provided by the SCB. They have yearly and monthly data. This will be transformed from monthly data into quarterly data using the period’s arithmetic average.

**GDP** – The data used is the Quarterly seasonally adjusted GDP measured from consumer data. This data represents the value of the goods and services consumed during a quarterly period.

**HOX index** - This is the Swedish residential real estate price index produced by KTH-Valueguard. It is updated on NASDAQ on a monthly basis.

**Household mortgage rate** – This data is taken from the SCB. The SCB records lending from Banks and subsidiary mortgage institutes. Every entry is calculated as the average household mortgage rate paid over a month, hence, data is produced month by month.

### 1.6 Limitations

By using a broad general macroeconomic approach this thesis will not investigate detailed implications of house price movements and their effect on the different statistical groups. This is a bachelor thesis with a limited timeframe. Therefore, this thesis will only focus on the estimated impact of the new amortization regulation relationship on house prices.

Limitations in terms of data, the KTH-Valueguard residential real estate index begins in 2005. This index differs from already existing data on Swedish housing prices because it includes both apartments and houses. This is an important feature of this data set and because of this, we limit our data sampling to the period 2005-2017. This limitation is a trade of between the need for a relatively large data set vs the underlying representation of the data itself. The inherent weakness of VAR models in general is that they can be densely parameterized\(^2\). With too few data points and too many variables, the degrees of freedom can get too low to produce a statistical significant estimation. Therefore there is a risk of overfitting and/or omitted

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\(^2\) Parameters to be estimated follow this equation: \(n(1 + n \times p)\) where \(p\) is the number of matrices, 1 represents the intercept vector, and \(n\) defines the number of variables. This equation represent the total number of parameters to be estimated for the whole system.
variable bias. That is why we limit our model to four specifically chosen variables based on extensive research done through our analysis of economic academic papers.

1.7 Disposition
A literature study will be presented in section 2. The literature studies section includes a presentation of the material used to form an understanding of the subject, why the choice of economic variables were chosen and a representation of the literature underlying our use of the SVAR model. Section 3 contains a description of the VAR/SVAR model and the statistical tests used to specify our model and which impulse response function this thesis will use. Section 4 details how the data was treated. Section 5 details the estimated model and the results and discussion. Section 6 contains a summary and conclusion.
2 Literature study
This chapter presents previous academic studies that concern the thesis major subjects. The studies were used as the theoretical foundation for this thesis.

2.1 Studies on amortization and house prices
The Swedish National Bank released a report (Swedish National Bank, 2014) where they investigated the impact of LTV amortization regulation on house prices using a dynamic stochastic general equilibrium (DSGE) model. They produced two scenarios - amortize the whole loan in 35 years and amortize 1-2 percent yearly. These two scenarios were then compared to a base scenario. The base scenario is a forecast without the amortization regulation. The first model produced a scenario where housing depreciating 12 percent during and after the transition phase. The model also simulated a reduction in consumption and GDP by 2.5 percent compared with the base scenario. The second scenario indicated that house prices were expected to depreciate at most by 5 percent during the transition period and a reduction in GDP growth would be less than 1 percent compared with the base scenario. Both these scenarios show slightly different short term effects after the implementation of amortization regulation, while the long term effect were estimated to be insignificant.

The Swedish Financial Authorities report *Consequences of a stricter amortisation requirement* (Finansinspektionen, 118, 2017) discussed the expected impact of the stricter mortgage loan regulation implemented on March 1, 2018. This was done by calculating the elasticities between debt service payments and debt, as well as debt service payments and house prices. They concluded that the growth rate of house prices would decrease with 1.5 percent nationwide, with the implementation of the 2018 LTI amortization requirement.

The Swedish Institute for European Policy Studies, published a report in 2014 on the effects of amortization on housing prices and consumption. Their analysis showed that the amortization terms can play an important role in households' borrowing capacity and housing prices. The effects of consumption by increased amortization will be lower if the new regulation only covered new mortgage loans. An increase in amortization can result in residential real estate price depreciation, which may spread the impact of amortization increase to the entire mortgage loan stock. This is because existing homeowners begin to cautiously save when their mortgage rates rise. They concluded three main factors affecting
house price in the event of new amortization regulation - interest rate, household debt ratios and amortization form. (Johansson, S et al, 2014).

The authors give a good example of how amortization effects using mathematics as follows: suppose there are two buyers: both have the same income, \( I \), and have the same expenses for residents and bonds, \( I_b \) and \( I_k \). Where buyer 1 only pays interest on his loan, while buyer 2 pays interest and amortization. They can most often add \( I_K \) on mortgage loans over a year. Housing purchaser 1’s annual loan expense is determined by the size of the loan, \( L_i \), and the interest rate, \( r \), on the mortgage:

\[ I_K = L_1 \times r \] (1)

For home buyers 1. And for home buyers 2:

\[ L_K = L_2(r + \alpha) \] (2)

When home buyers 1 and home buyers 2 have the same income, they can add as much to mortgage loans:

\[ L_1 \times r = L_2(r + \alpha) \] (3)

We redeem the quota between the amortization-free home buyer’s loan options in relation to the borrowing options for those who amortize:

\[ \frac{L_1}{L_2} = \frac{r + \alpha}{r} \] (4)

And because \( r > 0 \) and \( \alpha > 0 \) applies to \( L_1 / L_2 > 1 \). This means that home buyers can choose to pay more than those who amortize and we get higher housing prices.

2.2 Studies on factors Affecting House prices
K.Tsatsaronis et al (2004). They use VAR models, with five endogenous variables. The growth rate of GDP, the rate of inflation in consumer prices, the real short-term interest rate, the term spread - defined as the difference in yield between a long-maturity government bond and the short rate; and the growth rate in inflation adjusted bank credit. Their main result is
that inflation is the biggest propellant in house prices. There are two potential explanations for this finding, that there are a long lasting link between nominal interest rates inflation and housing prices. Long periods of high inflation followed by a period of decreasing price growth, can in the short term create asymmetric outcomes between house prices and long term determinants of real estate values.

Claussen (2013) investigated the Swedish house market using an Error Correction Model in his report “Are Swedish Houses Overpriced?” He concluded that an increase in real disposable income and a decrease in real mortgage rate can explain a substantial part of the price increase between 1996-2011.
3 Method
3.1 Time Series analysis
Time series analysis is a statistical technique that deals with time series data, or trend analysis. Nelson and Plosser (1982) argues that almost all macroeconomics time series have a unit root, and a time series is stationary if the mean and variance are constant over time. Many time series variance and mean value are not constant over time (Glynn et al, 2007) since they have a tendency to grow over time. For this thesis it was checked that the mean and variance are stationary and that they do not have a unit root. Since this thesis investigates house prices, and it fluctuates over time with no tendency to revert to a fixed mean it is therefore not stationary.

3.1.1 Stationarity
A variable is stationary if the mean and the variance of the variable is constant over time, and the covariance between two different values of the series will depend on the time interval, and not at the time when it was observed (Westerlund, 2005).

Mathematically, these can be expressed as follows:

\[ E(y_t) = (t) \]
\[ VAR(y_t) = (t) \]
\[ COV(y_{t_1}, y_{t_2}) = (t_1, t_2) \]

If these expressions are not fulfilled, it is non-stationary (Westerlund, 2005). Some variables are non-stationary, mainly because they increase continuously over time, their mean values and variations are not constant (Glynn et al, 2007). One can estimate stationary time series data using Ordinary Least Squares (OLS). It is thus important to check the time series to analyze if they are stationary or not, otherwise spurious regression can be obtained (Granger et al, 1974).

To investigate non-stationarity, one can transform them into stationary, which is done by differentiating the data. Then you can use the formula (Nielsen, 2005)

\[ \Delta y_t = y_t - I_{t-1} \quad (5) \]
Then it is called the differential stationary. One can also check trend stationarity. This means that the time series fluctuates around a linear trend, then subtracts the trend from the series.

3.1.2 Unit root
A time series that is stationary without differentiation, it is integrated by the order zero. If it is not stationary you differentiate the time serie until it becomes stationary. One can do this repeatedly until one gets a non-stationary time series to become stationary (Maddala et al, 1998). There are a variety of tests that can determine the occurrence of a unitary root in a time series. One of them is the Augmented Dickey-Fuller (ADF) test.

3.2 Structural Vector Autoregressive (SVAR) model
The structural vector autoregressive function was first introduced by Christopher A Sims in the 1980s as a critique of the standard practice of solving for a partial equilibrium and aggregating the results, from a set of equations. Sims put forth concerns for the disregard of the omitted interrelations of the partial equilibrium, but also that the specifications and restrictions of the systems of simultaneous equations. He suggested the use of multivariate vector autoregressive models (VARs) where each set of variables under investigation are regressed on a defined number of lags of itself and all the other variables under study. This would mean that all variables under consideration are endogenous. By doing this the VAR model captured the contemporaneous correlations of the variables in the instantaneous correlations structure of the error terms, given that variables are stationary, have zero mean error terms and constant variance in the error terms. Ordinary least square are used to estimate the Beta coefficients. The structural Vector Autoregressive model is a modification of the VAR model. By introducing restrictions on the VAR model i.e., the contemporaneous relationship between the variables, one can test the effect of structural innovation in the error term of variable $X$ on variable $Y$ and $Z$ by an iterative process (Amisano et al, 1997).

3.2.1 Augmented Dickey-Fuller (ADF) test
As mentioned in 3.1.2, one can check the occurrence of unit roots in time series using Augmented Dickey-Fuller (ADF) test, which is an expanded form of the Dickey-Fuller test (Dickey-Fuller, 1979).
The following hypotheses is:

\[ H_0 : \text{"have a unit root"} \]
\[ H_1 : \text{"do not have a unit root, and thus it is stationary"} \]

According to Dickey-Fuller (1981) the following regression is estimated by

\[ \Delta Y_t = \alpha + \beta_0 + Y_{t-1} + \sum_{i=1}^{L} \beta_i \Delta Y_{t-1} + U_T \quad (6) \]

There \( L \) is the number of lags, \( U_T \) is error term and is the first order of the difference term. The number of lags is determined by AIC and log likelihood and so on.

3.2.2 The number of lags
Akaike Information Criterion (AIC) are used to determine the optimal lag length for the reduced VAR model (M.Shrestha et al, 2017)

3.2.3 Akaike Information Criterion (AIC)
The Akaike information criteria (AIC) is an estimator of the relative quality of statistical models for a given set of data. For this thesis we are interested in what variable influences house prices and what variable affects those prices.

The advantage of AIC is that it balances the major disadvantages of the other models, for example, one cannot capture the true nature of variability in the output variable.

\[ AIC(M) = -2 \log((L(\{y\}) + 2P) \quad (7) \]

Where \( L \) is the log likelihood function of the parameters in the model. The factor 2 was introduced for historical reasons. The term \( 2P \) is the estimated bias. Denoting the parameter estimates as \( \theta \), given model \( M \) and a set of data \( y \) (M. Snipes et al, 2014)

3.2.4 The Johansen method
The concept of cointegration is developed from Engle and Granger (1987). Two variables are cointegrated if they share a common stochastic trend in the long-run. The general rule when combining two integrated variables is that their combination will always be integrated at the
lower of the two orders of integration. If there exists such linear combination of non-stationary variables that is stationary, cointegration between those variables exists. In order for two variables to be cointegrated they need to be integrated of the same order.

### 3.3 Model diagnostics
The general consensus regarding approximation and estimation of VAR-models is that one needs to apply a certain degree of “trial and error” (Gujarati et al, 2009). After the individual ADF and testing for the appropriate lag length of the model, one needs to test the inverted characteristic lag polynomial to be certain that all the roots are within the unit circle. This ensures us that the reduced VAR is stationary.

With the appropriate lag length, stationary individual variables and confirmation that all the roots of the inverted lag polynomial is within the unit circle, one proceed to check the structure of the residuals with the LaGrange multiplier test and heteroscedasticity test.

### 3.4 Lagrange multiplier (LM)-test
This test is looking for an autocorrelation in the error terms of a regression model. Set up this hypothesis:

\[
H_0: p_i = 0 \forall i \in \{1, \ldots, p\}
\]

\[
H_1: \text{otherwise}
\]

That is, there is no serial correlation of any order up to \( p \). (Woolridge, 2002)

### 3.5 Granger causality and heteroscedasticity
A distributed lag model is to provide evidence about the relationships between its variables. A Granger causality implies a correlation between the current value of one variable and the past values of others, it does not mean changes in one variable cause changes in another. This test is useful to see which variable causes the other to move, such leading variables are extremely useful for forecasting purposes. (Studenmund, 2017)

Heteroscedasticity briefly means that the variance of field terms is not constant; that is, when the value of independent variable, \( x \), increases, then the unexplained variation in dependent variable, \( y \), decreases or increases. If the spread is even, the opposite is homoscedasticity. This could cause OLS problems so the default errors for coefficients look bigger or smaller
than they actually should be. (Barretto, H et al, 2006) Heteroscedasticity is tested with an extension of White's heteroscedasticity test (White, 1980).

\[ H_0 = \text{"no heteroscedasticity"} \]
\[ H_1 = \text{"there is heteroscedasticity of some form"} \]

3.6 Model evaluation with MAE, Thiel and RMSE
To be able to determine our models forecasting volatility, this thesis will use different quality evaluation measures; Root mean square error (RMSE), Mean absolute error (MAE), Mean absolute percent error (MAPE) and Theil statistic.

RMSE aggregates the residuals into one single measure of predictive power. MAE is important to use the outcomes of the measures for comparison between different forecasting models tighter. (Wooldrigde, 2012) MAPE is built on MAE assumptions but represents the results in percent and shows how the model in question performs. The Theil statistic generally gives a more accurate picture because of the way it is measured, it identifies the share of inequality (Brooks, 2008).

3.7 Impulse response function (IRF)
Impulse response functions are useful for studying the interactions between variables in a vector autoregressive model. This is useful because it allows us to estimate what the system's output will look like in the time domain.

The calculation of the impulse response function is produced from the Wold representation\(^3\) of the reduced VAR. There are two major important points to consider regarding the production of the IRF - identification and orthogonalisation. Usually, the Cholesky decomposition is used to triangulase the coefficient matrix in the Wold representation, making it orthogonal. This is done to ensure that the impulses tracked by the IRF is uncorrelated. But by transforming the coefficient matrix one imposes restrictions on the contemporaneous relationships. This means that the ordering of the variables in the reduced VAR dictates which contemporaneous relationships are given significance in the model. Identification is the process of choosing a variable to shock and how that shock will be

\(^3\) For more information see Ronayne, 2011 and Amisano. G., Giannini. C. (1997)
transmitted through the system. This is done by ordering the variables in the VAR, and in doing so, imposing restrictions on the triangular orthogonal coefficient matrix (Ronayne, 2011).
4 Data
The model will be estimated on a data set that covers the period 2005 Quarter 1 to 2017 Quarter 4. The year 2005 is the first year we have residential real estate price data that includes apartments and houses. Before the year 2005 data only includes house price growth. This introduces a tradeoff and limitation that this thesis needs to justify. The implementing of a VAR model generally needs a larger data set due to exponential increase in parameters to be estimated. The inclusion of apartment data gives the shorter data set a more significant representation of the whole residential real estate market, this justifies the losses in degrees of freedom. The data is collected from statistic Sweden and Valueguard-KTH NASDAQ index.

The HOX SWE index consists of monthly data of Swedish house and apartment prices. It is produced by KTH-Valueguard and updated on a monthly basis. The price data is retrieved from Mäklarstatistik AB and Lantmäteriet. This data set have been transformed to quarterly data by taking the arithmetic average. The quarterly data have been converted to percent. (Nasdaq)

The GDP data set, are represented in seasonally adjusted quarterly percentage change. The calculations have been made by statistics Sweden (SCB).

Consumer price index (KPI) is a monthly index produced by statistic Sweden. The monthly index have been transformed to quarterly data by taking the arithmetic average. The quarterly data have then been transformed to percent. (SCB)

This thesis has used mortgage rate data on bank loans from SCB. This data set consists of quarterly data during the first 3 quarters in 2005. After Q3 2005 the data series consists of monthly data entries. The monthly entries have been transformed to quarterly data by taking the arithmetic average. Average quarterly KPI have been subtracted from the quarterly mortgage rate data to produce inflation adjusted quarter mortgage rates (SCB).

All the variables are originally in or have been converted to percent so that the data is denominated in the same unit. This conversion allows the results to have the same unit of measurement, which prepare the data set for regression, and make the conclusions easier to interpret. The data set range include the 2008 financial crisis which creates abnormal outliers.
as seen in figure 4.1 below. This can clearly be seen in the GDP and KPI graphs. The outliers increases the variance of the variables.

![Graphical representation of data](source: Statistic Sweden And Value guard)

Real mortgage rate have been differentiated once and the HOX index have been transformed to percent. This has been done even though the variables are stationary. The financial crisis can be seen as a significant structural break in the all time series. One interesting feature is the large percentage drop in residential real estate prices in 2017, taking place before the implementation of the March 1, 2018 amortization regulation. This will probably affect the explanatory power of our model when comparing the data to both a dynamic and static forecast evaluation later on.

The summary statistic in table A.1 in Appendix A show that HOX and GDP does not pass the Jarque-Bera test. This indicates that our data sample on HOX and GDP do not come from a normally distributed population. This kind of distributions is expected when dealing with a financial data series with specific instances where large value fluctuations increase the overall variance of the variable. KPI and RMR have acceptable Kurtosis and Skewness values, while
also passing the Jarque-bera test. The residual graphs in figure A.1 in Appendix A shows that the residuals have zero mean, and together with the LM test and the Heteroscedasticity test it can be concluded the error term is white noise. It is noted that even though two of four variables does not pass the Jarque-bera test, OLS is still BLUE due to the properties of our error terms.

Substantial correlation between KPI and Real mortgage rate can be seen in Table A.6 in Appendix A. This can introduce multicollinearity in the model. However, based on previous research which gives significance to both variables (K.Tsatsaronis et al, 2002, Claussen, 2013, Johansson, S et al, 2014) this thesis choose to incorporate both.
5 Result and discussion

5.1 Stationarity and unit root.
Two of our variables - real mortgage rate (RMRR) and HOX index display a trend. HOX produced statistically significant result of no unit root, after transforming it to percent and applying the ADF test with an intercept. Real mortgage rate produced statistically significant results of no unit root after taking the first difference and applying the ADF test with an intercept. Seasonally adjusted GDP produced statistically significant result displaying no unit root at level after applying the ADF test with an intercept. This thesis could not reject the null hypothesis when applying the ADF test on the KPI at level. After converting KPI to percent and applying the ADF test with an intercept, we achieved statistically significant results and could reject the null hypothesis, see table A.2. All the roots of the inverted lag polynomial lies within the unit circle see figure A.2 in Appendix A.

<table>
<thead>
<tr>
<th>Method</th>
<th>Statistic</th>
<th>Prob.**</th>
<th>Cross-sections</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null: Unit root (assumes common unit root process)</td>
<td>-3.42495</td>
<td>0.0003</td>
<td>4</td>
<td>197</td>
</tr>
<tr>
<td>Levin, Lin &amp; Chu*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Null: Unit root (assumes individual unit root process)</td>
<td>35.5894</td>
<td>0.0003</td>
<td>4</td>
<td>197</td>
</tr>
<tr>
<td>ADF - Fisher Chi-square</td>
<td>48.7960</td>
<td>0.0003</td>
<td>4</td>
<td>203</td>
</tr>
</tbody>
</table>

** Probabilities for Fisher tests are computed using an asymptotic Chi-square distribution. All other tests assume asymptotic normality.

TABLE A.2 Unit root test. For illustration how it looks like in a unit circle, see appendix figure A.2

5.2 Lag length criteria
The information criteria for optimal lag length is contingent on the number of observations. While the AIC and Final Prediction Error (FPE) are more appropriate when observations are less than 60, the Hannan-Quin (HQ) is more efficient when observations are above 120. Moreover, it remains the discretion of the authors to select the maximum lags which the adopted criterion for choosing optimal lags will use (Liew, 2004). It was suggested that the lag length should be set such that the VAR residuals are free of autocorrelation, even if this
implies longer lags than suggested by the information criteria. This thesis’s specified lag length is therefore four with the AIC when the test was run. See table A.3.

5.3 Granger causality
The pairwise Granger causality test verifies the choice of variables. Table 5.1 outlines the Granger causality relationship between the variables chosen.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>5 %</th>
<th>10 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPI</td>
<td>GDP, PHOX</td>
<td>RMRR</td>
</tr>
<tr>
<td>GDP</td>
<td>RMRR</td>
<td></td>
</tr>
<tr>
<td>PHOX</td>
<td>GDP, RMRR</td>
<td></td>
</tr>
<tr>
<td>RMRR</td>
<td>PHOX</td>
<td>KPI</td>
</tr>
</tbody>
</table>

*Table 5.1. Granger Causality with 5 to 10 % significance level. Full test results see Table A.4 in Appendix A.*

The paired causality tests show that there is a mutual statistical correlation between the interest rate and the KPI, interest rates and house prices. Only a one-way causality in relation to the rest of the variables. With Granger's causality test, it can be noted that the variables are appropriate when there is evidence of statistical correlation between these and house prices.

5.4 Johansen cointegration test
This thesis tested all joints with the Johansen cointegration test. If a cointegration was discovered, then the model combination stretched as a candidate for creating forecasts. The model did not show any cointegration. This thesis noted the maximum eigenvalue statistical results, which showed significant no significance on co-integrated equations. See table. A.5

5.5 Estimated model
To estimate the thesis model, we assumed that the estimated model does not change during the forecast period. In this respect, VAR differs from other macroeconomic models of prediction, focusing on exogenous variables, these models often have poor prognosis precision outside the sample (Brooks, 2008). But in a reduced VAR and SVAR model, the variables are treated as endogenous (Sims, 1980).

In figure 5.6 below, we represent the vectors and matrices that constitutes our reduced form VAR model.
\[ Y_t = \begin{pmatrix} Y_{t-1} \\ X_{t-1} \end{pmatrix} = \begin{pmatrix} (RMRR, PHOS, KPI, KPR, CD) \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} \] 

\[ \beta_0 = \begin{pmatrix} \beta_{0,1} \\ \beta_{0,2} \\ \beta_{0,3} \\ \beta_{0,4} \end{pmatrix}, \quad \beta_1 = \begin{pmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \beta_{1,3} \\ \beta_{1,4} \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} \beta_{2,1} \\ \beta_{2,2} \\ \beta_{2,3} \end{pmatrix}, \quad \beta_3 = \begin{pmatrix} \beta_{3,1} \\ \beta_{3,2} \\ \beta_{3,3} \end{pmatrix}, \quad \beta_4 = \begin{pmatrix} \beta_{4,1} \\ \beta_{4,2} \end{pmatrix} \]

\[ \epsilon_t = \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \end{pmatrix} \]

\[ \beta_3 = \begin{pmatrix} \beta_{3,10} \\ \beta_{3,14} \end{pmatrix}, \quad \beta_4 = \begin{pmatrix} \beta_{4,15} \\ \beta_{4,19} \end{pmatrix} \]

\[ \epsilon_t = \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \end{pmatrix} \]

Figur 5.6. Vectors and matrices in our VAR model.

This can be represented in a reduced form in figure 5.7. \( Y_t \) is a vector of values at time \( t \). \( \beta_0 \) is a vector of intercepts. \( \beta_{1-4} \) are coefficient matrices for the different lagged \( Y \) vectors. \( \epsilon_t \) are a vector of zero mean constant variance error terms. The reduced form Autoregressive model can be seen in equation 8.

\[ Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 Y_{t-3} + \beta_4 Y_{t-4} + \epsilon_t \]  

OLS is performed on each of the four equations. To enhance reliability, we perform tests on the residuals, autocorrelation, heteroscedasticity, cointegrated and the absolute value of the roots from the characteristic lag polynomial are located within the unit circle because they are then stationary.

5.6 Residuals White noise/LM test, heteroscedasticity and correlation
According to Whites heteroscedasticity, we could not reject the null hypothesis of heteroscedasticity and serial correlation, which meant that the results will be unbiased and constitutional to our specified lag, four. The residuals are not series correlated, which was satisfactory, because otherwise there is a risk of spurious regression. See table A.6 and A.7. The autocorrelation in figure A.3 in appendix A is within the acceptable range.

5.7 Model evaluation
The model evaluation is measuring the forecast and/or predictive capabilities of the model by calculating the forecast errors. This thesis has estimated the model for the period 2005 Q1-2015 Q4. The estimated VAR was then used to produce both a dynamic and static forecast for the period 2016 Q1-2017 Q4. The result were compared to the actual data in Figure A.5 and A.6 in appendix A. Both the static and dynamic forecasts produced a relative high RMSE for the HOX variable. This can be attributed to the sudden drop in the fourth quarter in 2017.
Removing the last quarter in 2017 significantly reduces the RMSE for HOX and improves the overall forecasting value for all variables. The MAE of all variables except HOX is below one. HOX MAE is close to two. Removing the last quarter in 2017 improves the overall MAE for all variables. The Theil statistic follows the same overall results as the RMSE and MAE for both the dynamic and static forecast. Excluding the last Quarter in 2017 significantly improves the overall predictive capacity of our VAR model, see table A.8-11 in appendix A.

5.8 PHOX, Impulse response function and results
The impulse response function test the movements of a variable from its mean given a non-zero error term. In a VAR model this non-zero error terms effect on the first variable is then transmitted throughout the system.

Figure A.8 Asymptotic 95% confides intervals - the red dotted line. And how it responded to Cholesky with one S.D innovation.

Figure A.9 Asymptotic 95% confides intervals - the red dotted line. And how it responded to Cholesky with one S.D innovation.
Figure A.10 Asymptotic 95% confidence intervals - the red dotted line. And how it responded to Cholesky with one S.D innovation

Figure A.8 shows that a shock to GDP has insignificant negative impact on house prices for approximately the whole periods. Figure A.9 shows the response to a shock to KPI has a positive but insignificant impact to house prices for the first period, it then decreases in period two, and then significantly increases in period 4 to 5, and finally becomes insignificant again and decrease. When we introduce an innovation in the RMRR equation house price growth reacts positive the first quarter. The second quarter is still positive but with decreasing growth. From quarter two to eight, house prices are expected to enter a downward trend in price growth. Values from the impulse response is only statistically significant at a 95% significant interval the first two quarters.
Figure A.11 Asymptotic 95% confidence intervals - the red dotted lines. Unconditional forecast of house price growth (2018Q1-2019Q4)

The unconditional dynamic forecast indicates that house prices will enter a phase with sporadic positive and negative price growth. The mean growth rate during the forecasted period is 1 percent per quarter. This is a reduction by 0.6 percentage points compared with the mean of our PHOX data sample. The compounded House price growth rate quarter to quarter during 2005Q2-2017Q4 is 1.58 percent. Extrapolating the mean quarterly percent growth rate from the forecasted period on the HOX index gives us a total price increase of 8 percent between 2018Q1-2019Q4 (see equation 9). This corresponds to a compounded percentage increase quarter to quarter of 0.99 percent (see equation 10). Both the mean growth rate and the compounded growth rate indicates decreasing house price growth rates post 2017. The results from the unconditional forecast are in line with the FIs (2017) report indicating lower growth rates post implementations of the 2018 amortization regulation.

\[ \text{HOX}_{2017\ Q4} \times \text{Mean}_{2018Q1-2019Q4}^{\text{HOX}} = \text{Estimated HOX}_{2019\ Q4} \]  
\[ \left( \frac{\text{Estimated HOX}_{2019\ Q4}}{\text{HOX}_{2017\ Q4}} \right)^{\frac{1}{8}} - 1 = 0.99 \]
5.9 Discussion
This thesis’s results stands in contradiction to previous research on amortization and house prices. Sweden's National Bank DSGE model suggested a price depreciation between 5-12 percent depending on the amortization Scenario (Swedish national Bank, 2014a). This thesis model does not produce a significant direct price decrease, but the impulse response shows the housing market entering a short term trend, with decreasing house price growth. The estimated downward trend agrees with the overall results from the Swedish financial Authorities report (2017) which concludes that the 2018 amortization requirement would reduce house price growth rate post implementation. The Unconditional forecast (see figure A.11) shows a drastic jump to positive growth rate from 2018Q1 followed by cyclical positive and negative price growth.

Our study is the opposite of what K.Tsatsaronis et al (2004) investigated. Theirs conclusions was that inflation influenced house prices most over a five-year period while we noted this in our study in two years. Their study was also done in 2004, and they examined the whole world but ignored major differences in the experience of individual countries. We took the financial crisis in 2008 in the thesis model and its drastic decline, and compared this data to previous year up until 2017, which can and did affect this thesis’ results.

The VAR model structures statistical relationships based on the intrinsic information contained in each data point. The estimated VAR becomes a representation of the total data sample. The unconditional forecast, and in some regard the IRF, is not produced with economic fundamentals as base, but more so, the statistical properties of the data set. Sims (1992) introduced the term price puzzle when discussing outcomes that contradicted economic intuition. Sims coined this term when applying a SVAR on the effects of monetary policy on inflation. Contrary to economic intuition, the impulse response function indicated an initial increase in inflation following a tightening of monetary policy. The response of HOX to a chock in the error term of d(RMRR) in figure A.10 indicate the inconclusiveness of the price puzzle. There are two ways of interpreting the results - did house prices appreciate because of the increase in the real mortgage rate, or, did the increase in mortgage rate reduce an even higher eventual price growth. This leads to a certain ambiguity in interpreting the results from the impulse response function. There are literature that propose different methods to mitigate the price puzzle problem in relation to forecasting models. Ben Bernanke suggest the use of Factor Augmented Vector Autoregressive (FAVAR) approach to deepen the analysis through
implementing principle component analysis to investigate the underlying driving forces between a larger dataset with more included variables (Bernanke, et al, 2004).

The unconditional forecast is a dynamic iteration of our estimated equations. The fundamental outcome indicated a period with decreasing growth house price growth rates. But how do we interpret this by economic theory when the model are estimated without any restrictions and a priori assumptions (except the choice of variables). The inherent information in the dataset indicates a reduced growth trend post 2017. This can be seen as forward expectation where by individual households and companies acts on information in anticipation of future changes in the housing market i.e. the implementation of the 2018 amortization requirement. The large price drop in the fourth quarter 2017 is of the same amplitude as the 2008 price drop. The two model evaluation forecasts (figure A.4-A.5 in appendix A) showed a reversion towards the mean in the fourth quarter 2017. The unconditional dynamic forecast starts from the price drop in 2017 Q4. This drop introduce a start from a negative growth rate resulting in a reduced mean over the forecasting period. In the estimation of the model “structural breaks” or outliners are a problem to be dealt with, but in the forecast it leads the results to a conclusion in line with previous research. Our estimated model could not forecast the short term price drop as the DSGE models used by Riksbanken (2014), but the inherent information in the data set given the large short term price adjustment in 2017 Q4 produce reasonable results regarding the future growth rates of the Swedish housing market. Given this thesis results we can assert the following:

1. The impulse response function results stand in contradiction to economic intuition i.e. the price puzzle problem.
2. The unconditional forecast indicates that the housing market will enter a period with slower growth post 2017.
3. The use of a vector autoregressive model can give meaningful results with regard to trend forecasts but with regard to precise statements as anticipating drastic price depreciation, it falls short.

The vector autoregressive model results is strictly based on the dataset used. Given a different dataset, choice of variables or forecast period we would get different results, hence our results should be interpreted with caution. We found that it is possible to make good forecasts with a reduced VAR model and thus without any underlying economic assumptions except the
choice of variables to include. A trap you can easily get when VAR models are developed is that you include too many variables or layers, so-called "over-fitting", but still no guarantee that the forecasts will be good. This can be reduced using the Bayesian Vector Autoregressive (BVAR) so as to limit the number of parameters. Another thing is that trace statistic indicated cointegration in our model. Thus, it can be solved with the proprietary system Vector error correction model (VECM). The VECM introduces error correction features to a standard VAR model, taking into account the cointegration relationship in the model.
6 Conclusion
This thesis main objective was to forecast the effect of the 2018 amortization regulations effect on house prices development. For this we used two methods - impulse response function and unconditional forecast. Even though this thesis did not reach any solid conclusions regarding the impact of the mars 1, 2018 amortization requirement on the broader economy. This thesis could deduce some reasonable results regarding the future price growth of the Swedish residential real estate market.

The impulse response for the residential real estate market with regard to our chosen variables only produced statistically significant outcome the first two quarters post choking the error term of real mortgage rate. This can be seen as results that stand in contradiction to economic intuition, indicating the price puzzle problem. Introducing structural innovations to error terms of GDP and KPI did not produce statistically significant outcomes. The unconditional forecast indicating an overall reduced house price growth compared with the data set mean used to produce the forecasting model. This is supported by Johansson, et al (2014) and Swedish economic research institutes (2015) economic reasoning regarding the ripple effect, how a specific macro prudential regulation targeting a certain group will affect the broader market.

With this thesis final remarks the authors would like to point out the uncertainties with forecasting. All forecasts should be seen with uncertainty. The future is impossible to predict regardless of using a complicated model or a simple VAR model. At a really long term, the forecast loses its’ predictive.
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Appendix A. Figures

Figure A.1 Var residuals

Table A.2 Graphical data representation (source: Statistic sweden, Valueguard. Calculations have been made by SCB, Valueguard and the writers of this thesis)
Figure A.2 Unit root illustration

Figure A.3 Autocorrelation
Figure A.4 Dynamic Forecast

Figure A.5 Static forecast
## Appendix A. Tables

### Table A.1 summary statistic

<table>
<thead>
<tr>
<th></th>
<th>PHOX</th>
<th>RMRR</th>
<th>KPI</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.628094</td>
<td>2.128039</td>
<td>1.131765</td>
<td>0.547059</td>
</tr>
<tr>
<td>Median</td>
<td>1.937024</td>
<td>2.330000</td>
<td>0.930000</td>
<td>0.700000</td>
</tr>
<tr>
<td>Maximum</td>
<td>6.239447</td>
<td>4.510000</td>
<td>4.270000</td>
<td>2.400000</td>
</tr>
<tr>
<td>Minimum</td>
<td>-8.227568</td>
<td>-0.370000</td>
<td>-1.430000</td>
<td>-3.700000</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.990560</td>
<td>1.140866</td>
<td>1.272187</td>
<td>1.060444</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.009196</td>
<td>-0.278424</td>
<td>0.459062</td>
<td>-1.793640</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.583774</td>
<td>2.321483</td>
<td>2.628275</td>
<td>7.740269</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>15.414190</td>
<td>1.637240</td>
<td>2.094901</td>
<td>75.09481</td>
</tr>
<tr>
<td>Probability</td>
<td>0.000450</td>
<td>0.441040</td>
<td>0.352590</td>
<td>0.000000</td>
</tr>
<tr>
<td>Sum</td>
<td>63.03261</td>
<td>108.5300</td>
<td>57.72000</td>
<td>27.90000</td>
</tr>
<tr>
<td>Sum Sq. Dev.</td>
<td>447.1723</td>
<td>65.07880</td>
<td>80.92294</td>
<td>56.22706</td>
</tr>
<tr>
<td>Observations</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
</tr>
</tbody>
</table>

Table A.1 *summary statistic*

### VAR Lag Order Selection Criteria

Endogenous variables: PHOX D(RMRR) KPI GDP
Exogenous variables: C
Date: 05/18/18  Time: 11:52
Sample: 2005Q1 2017Q4
Included observations: 47

<table>
<thead>
<tr>
<th>Lag</th>
<th>LcgL</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-202.6740</td>
<td>NA</td>
<td>2.333589</td>
<td>12.19368</td>
<td>12.36635</td>
<td>12.25015</td>
</tr>
<tr>
<td>1</td>
<td>-134.2574</td>
<td>229.5105</td>
<td>0.019564</td>
<td>7.415208</td>
<td>8.202565*</td>
<td>7.711474</td>
</tr>
<tr>
<td>2</td>
<td>-135.9163</td>
<td>29.65800</td>
<td>0.018311</td>
<td>7.315566</td>
<td>8.732720</td>
<td>7.948663</td>
</tr>
<tr>
<td>3</td>
<td>-121.5925</td>
<td>20.72370</td>
<td>0.020132</td>
<td>7.386916</td>
<td>9.433889</td>
<td>6.157205</td>
</tr>
<tr>
<td>4</td>
<td>-82.04334</td>
<td>56.48833*</td>
<td>0.067990*</td>
<td>6.384323*</td>
<td>9.081632</td>
<td>7.392124*</td>
</tr>
</tbody>
</table>

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

TABLE A.3 *Lags order selection criteria.*
TABLE A.4 Granger Causality

<table>
<thead>
<tr>
<th>Null Hypothesis:</th>
<th>Obs</th>
<th>F-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPI does not Granger Cause GDP</td>
<td>48</td>
<td>3.75731</td>
<td>0.0112</td>
</tr>
<tr>
<td>GDP does not Granger Cause KPI</td>
<td></td>
<td>1.53755</td>
<td>0.2103</td>
</tr>
<tr>
<td>PHOX does not Granger Cause GDP</td>
<td>47</td>
<td>5.68816</td>
<td>0.0011</td>
</tr>
<tr>
<td>GDP does not Granger Cause PHOX</td>
<td></td>
<td>2.06978</td>
<td>0.1040</td>
</tr>
<tr>
<td>RMRR does not Granger Cause GDP</td>
<td>48</td>
<td>0.68770</td>
<td>0.6046</td>
</tr>
<tr>
<td>GDP does not Granger Cause RMRR</td>
<td></td>
<td>2.62419</td>
<td>0.0452</td>
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<tr>
<td>PHOX does not Granger Cause KPI</td>
<td>47</td>
<td>1.13195</td>
<td>0.3561</td>
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<tr>
<td>KPI does not Granger Cause PHOX</td>
<td></td>
<td>6.63360</td>
<td>4E-05</td>
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<tr>
<td>RMRR does not Granger Cause KPI</td>
<td>48</td>
<td>2.45575</td>
<td>0.0616</td>
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<tr>
<td>KPI does not Granger Cause RMRR</td>
<td></td>
<td>5.22955</td>
<td>0.0018</td>
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<tr>
<td>RMRR does not Granger Cause PHOX</td>
<td>47</td>
<td>7.82767</td>
<td>0.0001</td>
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<td>PHOX does not Granger Cause RMRR</td>
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<td>3.22955</td>
<td>0.0224</td>
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</tbody>
</table>

Sample (adjusted): 2005Q1 2017Q4
Included observations: 49 after adjustments
Trend assumption: Linear deterministic trend
Series: PHOX DR(RMRR) KPI GDP
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesis (No. of CE(s))</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>Critical Value</th>
<th>Prob. **</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.401566</td>
<td>52.64004</td>
<td>47.86613</td>
<td>0.0174</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.256440</td>
<td>26.72273</td>
<td>26.59767</td>
<td>0.0044</td>
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<tr>
<td>At most 2</td>
<td>0.205251</td>
<td>15.95005</td>
<td>15.66471</td>
<td>0.0554</td>
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<tr>
<td>At most 3 *</td>
<td>0.097073</td>
<td>4.743111</td>
<td>3.841466</td>
<td>0.0234</td>
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</tbody>
</table>

Trace test indicates 1 cointegrating eq(s) at the 0.05 level  
* denotes rejection of the hypothesis at the 0.05 level  
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesis (No. of CE(s))</th>
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<th>Max-Eigen Statistic</th>
<th>Critical Value</th>
<th>Prob. **</th>
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<tbody>
<tr>
<td>None</td>
<td>0.401566</td>
<td>23.81820</td>
<td>23.59434</td>
<td>0.1486</td>
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<tr>
<td>At most 1</td>
<td>0.256440</td>
<td>13.92609</td>
<td>13.9162</td>
<td>0.3903</td>
</tr>
<tr>
<td>At most 2</td>
<td>0.205251</td>
<td>10.45194</td>
<td>10.41680</td>
<td>0.1839</td>
</tr>
<tr>
<td>At most 3 *</td>
<td>0.097073</td>
<td>4.743111</td>
<td>3.841466</td>
<td>0.0234</td>
</tr>
</tbody>
</table>

Max-eigenvalue test indicates no cointegration at the 0.05 level  
* denotes rejection of the hypothesis at the 0.05 level  
**MacKinnon-Haug-Michelis (1999) p-values

Table A.5 Johansen cointegration test with linear deterministic trend.
VAR Residual Serial Correlation LM Tests

Date: 05/10/19  Time: 11:53
Sample: 200501-201704
Included observations: 47

Null hypothesis: No serial correlation at lag h

<table>
<thead>
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<tbody>
<tr>
<td>1</td>
<td>16.96282</td>
<td>16</td>
<td>0.3880</td>
<td>1.078024</td>
<td>(16, 70.9)</td>
<td>0.3917</td>
</tr>
<tr>
<td>2</td>
<td>12.84780</td>
<td>16</td>
<td>0.6839</td>
<td>0.794554</td>
<td>(16, 70.9)</td>
<td>0.8866</td>
</tr>
<tr>
<td>3</td>
<td>11.57473</td>
<td>16</td>
<td>0.7727</td>
<td>0.709046</td>
<td>(16, 70.9)</td>
<td>0.7749</td>
</tr>
<tr>
<td>4</td>
<td>17.15097</td>
<td>16</td>
<td>0.3759</td>
<td>1.031346</td>
<td>(16, 70.9)</td>
<td>0.3796</td>
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</tbody>
</table>

Null hypothesis: No serial correlation at lags 1 to h

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>16.96282</td>
<td>16</td>
<td>0.3880</td>
<td>1.078024</td>
<td>(16, 70.9)</td>
<td>0.3917</td>
</tr>
<tr>
<td>2</td>
<td>25.84889</td>
<td>32</td>
<td>0.7703</td>
<td>0.778270</td>
<td>(32, 71.7)</td>
<td>0.7814</td>
</tr>
<tr>
<td>3</td>
<td>30.36795</td>
<td>40</td>
<td>0.9300</td>
<td>0.734845</td>
<td>(40, 59.0)</td>
<td>0.9645</td>
</tr>
<tr>
<td>4</td>
<td>62.74208</td>
<td>64</td>
<td>0.5211</td>
<td>0.903089</td>
<td>(64, 45.3)</td>
<td>0.6503</td>
</tr>
</tbody>
</table>

*Edgeworth expansion corrected likelihood ratio statistic.

**TABLE A.2** Residuals White noise/LM test

<table>
<thead>
<tr>
<th>Correlation</th>
<th>KPI</th>
<th>GDP</th>
<th>HOX</th>
<th>RMRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPI</td>
<td>1.000000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>-0.210127</td>
<td>1.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HOX</td>
<td>-0.110279</td>
<td>0.156253</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>RMRR</td>
<td>-0.683493</td>
<td>-0.118250</td>
<td>-0.572729</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

**TABLE A.6** Correlation
### Table A.7 Heteroscedasticity

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>res1*res1</td>
<td>0.567140</td>
<td>0.573218</td>
<td>0.94051</td>
<td>25.65556</td>
<td>0.7339</td>
</tr>
<tr>
<td>res2*res2</td>
<td>0.785387</td>
<td>1.565755</td>
<td>0.1800</td>
<td>36.75220</td>
<td>0.2560</td>
</tr>
<tr>
<td>res3*res3</td>
<td>0.751803</td>
<td>1.325622</td>
<td>0.2933</td>
<td>35.33077</td>
<td>0.3133</td>
</tr>
<tr>
<td>res4*res4</td>
<td>0.516371</td>
<td>0.467119</td>
<td>0.9630</td>
<td>24.26045</td>
<td>0.8345</td>
</tr>
<tr>
<td>res2*res1</td>
<td>0.782215</td>
<td>1.571362</td>
<td>0.1854</td>
<td>36.76410</td>
<td>0.2576</td>
</tr>
<tr>
<td>res3*res1</td>
<td>0.757838</td>
<td>1.369143</td>
<td>0.2708</td>
<td>35.61940</td>
<td>0.3010</td>
</tr>
<tr>
<td>res4*res2</td>
<td>0.777170</td>
<td>1.525937</td>
<td>0.2018</td>
<td>35.52729</td>
<td>0.2664</td>
</tr>
<tr>
<td>res4*res1</td>
<td>0.568842</td>
<td>0.521417</td>
<td>0.8698</td>
<td>27.58158</td>
<td>0.6899</td>
</tr>
<tr>
<td>res4*res2</td>
<td>0.606975</td>
<td>1.068276</td>
<td>0.5193</td>
<td>32.75763</td>
<td>0.4266</td>
</tr>
<tr>
<td>res4*res3</td>
<td>0.657409</td>
<td>0.839830</td>
<td>0.6716</td>
<td>30.90196</td>
<td>0.5220</td>
</tr>
</tbody>
</table>

### Table A.8 Dynamic forecast evolution

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inc. obs.</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
<th>Theil</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>8</td>
<td>0.721230</td>
<td>0.622935</td>
<td>144.4949</td>
<td>0.472338</td>
</tr>
<tr>
<td>KPI</td>
<td>8</td>
<td>0.320817</td>
<td>0.205325</td>
<td>29.30568</td>
<td>0.114599</td>
</tr>
<tr>
<td>PHOX</td>
<td>8</td>
<td>2.486745</td>
<td>1.959550</td>
<td>191.6313</td>
<td>0.423363</td>
</tr>
<tr>
<td>RMRR</td>
<td>8</td>
<td>0.289327</td>
<td>0.242415</td>
<td>90.34527</td>
<td>0.197920</td>
</tr>
</tbody>
</table>

RMSE: Root Mean Square Error  
MAE: Mean Absolute Error  
MAPE: Mean Absolute Percentage Error  
Theil: Theil Inequality Coefficient
### Table A.9 Dynamic forecast evolution

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inc. obs.</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
<th>Theil</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>7</td>
<td>0.794216</td>
<td>0.676671</td>
<td>195.9243</td>
<td>0.500174</td>
</tr>
<tr>
<td>KPI</td>
<td>7</td>
<td>0.506754</td>
<td>0.461393</td>
<td>32.05909</td>
<td>0.159749</td>
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<tr>
<td>PHOX</td>
<td>7</td>
<td>1.321526</td>
<td>1.167531</td>
<td>659.7255</td>
<td>0.273641</td>
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<tr>
<td>RMRR</td>
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<td>0.463244</td>
<td>0.330906</td>
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<td>0.305905</td>
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</tbody>
</table>

RMSE: Root Mean Square Error  
MAE: Mean Absolute Error  
MAPE: Mean Absolute Percentage Error  
Theil: Theil inequality coefficient

### Table A.10 Static forecast evolution

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inc. obs.</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
<th>Theil</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>8</td>
<td>0.956478</td>
<td>0.848393</td>
<td>135.7824</td>
<td>0.564587</td>
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<tr>
<td>KPI</td>
<td>8</td>
<td>0.547668</td>
<td>0.457927</td>
<td>32.40360</td>
<td>0.186963</td>
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<tr>
<td>PHOX</td>
<td>8</td>
<td>2.646102</td>
<td>1.917593</td>
<td>612.0855</td>
<td>0.269337</td>
</tr>
<tr>
<td>RMRR</td>
<td>8</td>
<td>0.511563</td>
<td>0.408101</td>
<td>109.4370</td>
<td>0.350765</td>
</tr>
</tbody>
</table>

RMSE: Root Mean Square Error  
MAE: Mean Absolute Error  
MAPE: Mean Absolute Percentage Error  
Theil: Theil inequality coefficient

### Table A.11 Static forecast evolution

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inc. obs.</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
<th>Theil</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>7</td>
<td>0.528517</td>
<td>0.435491</td>
<td>69.78274</td>
<td>0.359218</td>
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<tr>
<td>KPI</td>
<td>7</td>
<td>0.282416</td>
<td>0.250031</td>
<td>24.26149</td>
<td>0.104240</td>
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<tr>
<td>PHOX</td>
<td>7</td>
<td>1.360393</td>
<td>1.217564</td>
<td>71.9741</td>
<td>0.276272</td>
</tr>
<tr>
<td>RMRR</td>
<td>7</td>
<td>0.255122</td>
<td>0.217920</td>
<td>60.39044</td>
<td>0.168362</td>
</tr>
</tbody>
</table>

RMSE: Root Mean Square Error  
MAE: Mean Absolute Error  
MAPE: Mean Absolute Percentage Error  
Theil: Theil inequality coefficient