

A Brief Introduction to
Transcendental Phenomenology
and
Conceptual Mathematics

(En kort introduktion till transcendental fenomenologi och konceptuell matematik)

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Abstract

By extending Husserl's own historico-critical study to include the conceptual mathematics of more contemporary times – specifically category theory and its emphatic development since the second half of the 20th century – this paper claims that the delineation between mathematics and philosophy must be completely revisited. It will be contended that Husserl's phenomenological work was very much influenced by the discoveries and limitations of the formal mathematics being developed at Göttingen during his tenure there and that, subsequently, the rôle he envisaged for his material *a priori* science is heavily dependent upon his conception of the definite manifold. Motivating these contentions is the idea of a mathematics which would go beyond the constraints of formal ontology and subsequently achieve coherence with the full sense of transcendental phenomenology. While this final point will be by no means proven within the confines of this paper it is hoped that the very fact of opening up for the possibility of such an idea will act as a supporting argument to the overriding thesis that the relationship between mathematics and phenomenology must be problematised.

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PROLEGOMENA

§1 Introduction

Thanks to the arduous work of Husserl and his disciples, phenomenology today finds itself as gatekeeper to a veritable goldmine of scientific results regarding concrete experience. Following on from this comes a relatively recent trend, known as the naturalisation of phenomenology, which attempts to integrate this data into modern scientific research – largely in the field of cognitive science. There is yet another trend, represented by the likes of Claire Ortiz Hill, Jairo José da Silva, and Mirja Hartimo for example, that looks to further investigate Husserlian phenomenology's connections with the philosophy of mathematics more generally. What these trends have in common is that they both – in their own separate ways – attempt to tread the rather obscure line between phenomenology and mathematics. In this paper it will be argued that there is perhaps a third way to investigate this perimeter, one which may be viewed as a heretical chimera of the first two, and which ultimately amounts to the radicalisation of phenomenology by way of contemporary mathematics. What is required then is a historico-critical investigation that will attempt to come to terms with phenomenology's *telos* by taking a closer look at its relationship with mathematics. This contention is announced here as part of a series of introductory remarks but it is also worth mentioning that the thesis itself claims to be no more than a prolegomenon for future investigations. In the spirit of the tradition of transcendental philosophy, this paper will take as its modest aim the raising of phenomenological problems that will in turn require further investigation. The conclusion of this thesis will, if all succeeds as planned, point to the possibility of a transcendentalisation of a specific special science – category theory – and to an associated mathematisation of phenomenology. This would mean, more specifically, the imbuing of category theory with transcendental sense by employing it to investigate problems of intentional constitution and, as a result, opening up certain phenomenological regions to mathematics. It is hoped that by awakening these possibilities the paper's overriding thesis, that the delineation between mathematics and philosophy must be considered anew, will be significantly reinforced. This is by no means an uncomplicated matter and this study's conclusions will no doubt leave the reader with more questions than answers. One such question would undoubtedly

be: if a transcendental mathematics is truly possible, and category theory is such an obvious fit for such a science, why is there no existing literature on the topic? Gilbert T. Null and Roger A. Simons investigate the extension of set-theoretical based mathematics to transcendental problems of course, and Sebastjan Vörös sensibly highlights that a phenomenologisation of the natural sciences would need to be undertaken if a naturalisation of phenomenology were to be taken seriously, but in neither case is there a turn to conceptual mathematics for a solution.¹ On the other side of the coin there is undoubtedly academic interest in the benefits of category-theoretical methods in the modelling of consciousness, exemplified by the work of Z. Arzi-Gonczarowski and D. Lehmann, but this – unsurprisingly – shows no concern for questions relating to transcendental subjectivity.² So why has no one put two and two together (so to speak)? On this it is only possible to speculate, but perhaps the answer to this question will in fact go some way towards supporting this paper’s thesis. Operating with pre-given notions of the disparate natures of mathematics and transcendental phenomenology, the possibility of a combination of the two seems to have been obscured from view. It is hoped that, by turning to a historico-critical approach, while it will admittedly not be possible to confirm the idea of a transcendental mathematics, the need to revisit the division of labour between mathematics and phenomenology will remain unequivocal.

§2 The task at hand

The task to be undertaken is not one of saving the objective sciences from a transcendental *epochē* – that instead falls upon those advocating for the naturalisation of phenomenology. There is no hope harboured in this paper of sparing formal mathematics the parenthesizing to which it is due. The task is not to explicate Husserl’s latent importance to the philosophy of mathematics. The aim is rather to suggest that there may be a mathematics that, as non-naive, distinguishes itself from

¹ Gilbert T. Null and Roger A. Simons, “Manifolds, Concepts and Moment Abstracta,” in *Parts and Moments: Studies in Logic and Formal Ontology*, ed. Barry Smith (München: Philosophia Verlag, 1982); Sebastjan Vörös, “The Uroboros of Consciousness: Between the Naturalisation of Phenomenology and the Phenomenologisation of Nature,” *Constructivist Foundations* 10, no. 1 (2014): 96–104.

² Z. Arzi-Gonczarowski and D. Lehmann, “From Environments to Representations— a Mathematical Theory of Artificial Perceptions,” *Artificial Intelligence* 102, no. 2 (July 1, 1998): 187–247.

formal mathematics and, subsequently, earns itself the title of transcendental science. It is a mathematics immune to the reduction which piques the interest, and the possibility of such a mathematics that motivates this paper's thesis. As the study is to be a historico-critical one, the aim will be to shed light on the true sense of Husserlian phenomenology through the contemporary developments of mathematics. The task then becomes one of explicating the implicit importance of mathematics to phenomenology and this comes of course with an associated risk of absurdity. Admittedly, being attempted here is nothing more than the exploration of a possible kinship between transcendental phenomenology and mathematics (in the guise of category theory). It could easily be misconstrued then that the task is one of applying formal mathematics to philosophy or of turning phenomenology into a positive science. It will hopefully be possible to show that this is not the case. It is also perhaps necessary to emphasise that any talk of the mathematisation of phenomenology does not mean to suggest that all transcendental problems are to fall within the region of conceptual mathematics, that is to say that it is not being proposed that there is no room left for transcendental psychology in the investigation of subjectivity. Rather, in problematising the division of labour between phenomenology and mathematics, the task is to show that there are perhaps some areas, such as the critique of logical reason, which would be best delegated to a phenomenological mathematics and that by working in concert with transcendental psychology, mathematics may be able to help phenomenology realise its full sense.

After, in chapter I, assessing the critical situation in which Husserl found himself, a critique of the relationship between formal mathematics and phenomenology will be carried out in chapter II. This will include an attempted elucidation of the definitive rôle that Husserl's concept of the definite manifold plays in defining the domain of transcendental phenomenology. Chapter III will then be devoted to addressing, and problematising, the critique of logical reason as outlined by Husserl, with the aim of opening up for the possibility of mathematics after the phenomenological-transcendental attitude has been invoked and the objective sciences have been parenthesized by an all-embracing transcendental *epochē*. With the possibility of a transcendental mathematics hopefully now being promoted somewhat – or at the very least not being ruled out – there will be, in chapter IV, a focus on establishing the possibility of category theory as a candidate for the new branch of transcendental

science which this paper proposes. All this is carried out in the hope of establishing that the division between mathematics and phenomenology must be re-thought.

§3 On method

Upon completion of Husserl's *The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy* (henceforth *Crisis*) the reader is left standing at a crossroads of sorts: even if they find themselves utterly convinced by the conclusions of the text, and consequently also of its methodology, they are forced to choose which of these two will be taken as premise for any future philosophical inquiries. But how can this be? Surely with a set of rigorous scientific results at hand it is guaranteed that any subsequent reiteration of the method in question will of necessity yield the same outcome, otherwise the hypotheses would be falsified and it would be necessary to start again from scratch. The reason for this rather unique situation is that the method employed by Husserl is not a historical one, but rather a "teleological-historical" or "historico-critical" one.³ When the sciences are studied in this fashion it is realised that without any understanding of their beginnings no understanding can be reached as concerns their inherent meaning. At the same time, by returning directly to their origin there will be no understanding of the way in which their sense reveals itself throughout the development of the science in question. So, in order to reveal their teleological sense, a method must be employed that allows for the moving back and forth throughout history in a "zigzag pattern".⁴ Now, obviously, any study carried out in this manner cannot extend beyond its contemporary situation in terms of the historical data it has at its command. That is to say that Husserl could only start and end with the sciences of his time. However, anyone alive today stands at a point in history and, more specifically, armed with a manifold of pre-given mathematics that were of essential necessity inaccessible to Husserl. So in other words the opportunity of understanding Husserl in a way that he could never have understood himself presents itself.⁵ This

³ Edmund Husserl, *The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy*, trans. David Carr (Evanston: Northwestern University Press, 1970), 3; Edmund Husserl, *Formal and Transcendental Logic*, trans. Dorion Cairns (The Hague: Martinus Nijhoff, 1969), §100.

⁴ Husserl, *Crisis*, 58.

⁵ *Ibid.*, 73.

means that if Husserl's historico-critical methodology is inherited then the results of his study must be subsumed into any more contemporary investigation: past philosophers cannot be taken at their word and no one can be allowed – not even Husserl himself – to escape this rule of thumb.⁶ The method will involve performing a critique upon Husserl's phenomenology – in all its glorious failings and shortcomings – while at the same time being an attempt to reveal its full sense. What, in *Crisis*, Husserl does to Kant, Hume, Galileo etc. an attempt will be made here to do to Husserl himself.

It could be argued that this choice of method conveniently alleviates any responsibilities felt by a more traditional historical investigation. Formulated in a perhaps slightly more crass fashion: it permits the picking and choosing of historical data. This could be rebutted by saying that, while this may very well be true, the method also comes with the burden of ascribing sense to what otherwise might have been disregarded as nonsense or absurdity. This is not something that is necessarily required of the historian. With this in mind it is hoped that the investigation will go somewhat towards justifying the methodological choices made but – at the same time – the fruitfulness belonging to any future critique of the historico-critical method employed are appreciated.

§4 A terse note on language

The German term “*Mannigfaltigkeit*”, depending on the text and the translator, has been rendered as either “multiplicity” or “manifold”. In this text the choice has been made to follow the lead of David Carr, Dallas Willard and J.N. Findlay in preferring to use the term “manifold”. “*Unsinn*” will be rendered as “nonsense” or, where clarification is needed, “senseless” while “*Widersinn*” will be rendered as “absurd” or in some cases, in the interest of emphasis, “countersense”. When appearing in adjectival form “categorial” will be related to “category” in the sense employed by Husserlian phenomenology.

⁶ Ibid.

I. A CRITICAL SITUATION

In the opening pages of *Crisis* Husserl laments at the state in which he finds the sciences of his day. At that point in history science was of course booming in terms of its results so it is important to note that the crisis Husserl is interested in concerns not the productivity of the sciences but rather the questionability of their “genuine scientific character”.⁷ In this first chapter an attempt will be made to carry out an assessment of the nature of the crisis of which Husserl speaks.

§5 The mathematisation of nature and the naturalisation of the world

For Husserl the nomological sciences, that is to say the exact sciences driven by the marvel of modern mathematics, are to be both admired and admonished. It is undeniable that the formulae of positive science present civilisation with the quite remarkable ability of making systematically ordered predictions but it is important to be wary of the transformation of meaning which has at the same time, as part of the ongoing development of science and the production of its realm of objectivity, inevitably taken place. Euclid is responsible, in Husserl’s view, for laying forth the ideal of exactness which would in the modern period consume the sciences. This he achieved with his axiomatisation of geometry which, at the same time as it set about structuring the theme of geometry under a finite collection of homogenous laws, led “almost automatically...to the emptying of its meaning”.⁸ While bringing to light a whole range of universal tasks and instigating the idea of a “systematically coherent deductive theory” Euclid allowed geometry to transgress beyond its traditional focus on the practical tasks of everyday life.⁹ What was once a science of the very practical requirements of praxes, such as surveying, had – thanks to a collection of seemingly innocuous axioms – been transformed into a science of infinite tasks. It was then only a matter of time until, just like Euclidean geometry succeeded in idealising spatio-temporal shapes, Galileo succeeded in transforming the totality of nature into a mathematical manifold. From this point in history onwards the sciences were able to treat nature as a totality determined by the exact laws of causality and, in this way, were able to make valuable predictions with an ever-increasing degree of precision.

⁷ Ibid., 3.

⁸ Ibid., 44.

⁹ Ibid., 21ff.

But while nature can indeed be interpreted as a mathematical hypothesis, one with a track record of astonishing levels of success of course, it would be absurd to believe that the world – as “a world of knowledge, a world of consciousness, a world with human beings” – could also be understood as a complete system of laws which the positive sciences are able to explain by way of their infinite task of deduction.¹⁰ That is to say that, despite the fact that they are most definitely deserving of the utmost respect and admiration, the nomological sciences cannot be allowed to extend their region beyond the methodological framework to which they are essentially bound. The crisis experienced by Husserl then is not the mathematisation of nature but rather, more specifically, the naturalisation of the world. So Galileo’s nature, which has as its “mathematical index” the idealised shapes of Euclidean geometry, is an objectification of the concrete world of immediate experience and as such should not be confused with the very world which it aims to idealise.¹¹ This however is the very crisis which modern science has undergone as it erroneously takes the model for the modelled and, as a result, loses touch with the world it set out to explain.

§6 A mathematical crisis – not a crisis of mathematics

In his elucidation of the role of mathematics in this crisis of science Jairo José da Silva believes that, while it is true that Husserl stood in awe of the achievements of modern mathematics, he at the same time feared for them “degenerating” into mere technique.¹² But slightly opposed to this it is perhaps more important to emphasise that what Husserl is recounting, while it may be a crisis brought about by mathematics, is not a crisis of mathematics itself. For Husserl, the fact that material mathematics has through the course of history transformed into formal mathematics, and that subsequently the theory of manifolds has become a technique devoid meaning, is both legitimate and necessary.¹³ What Husserl is truly concerned about is not the state of mathematics but rather the “decapitation” of philosophy that the success of the nomological sciences has given rise to.¹⁴ It is objectivism (as opposed

¹⁰ Ibid., 265.

¹¹ Ibid., 37.

¹² Jairo da Silva, “Mathematics and the Crisis of Science,” in *The Road Not Taken: On Husserl’s Philosophy of Logic and Mathematics* (London: College Publications, 2013), 347.

¹³ Husserl, *Crisis*, 47.

¹⁴ Ibid., 9.

to transcendentalism) within philosophy that worries Husserl, he is simply perturbed by philosophy's seeming will to meet the exacting standards of the objective sciences. This pattern is so disturbing for Husserl because philosophy, which by all rights should be the first science from which all other sciences stem, is subsequently reduced to just one among many of the special sciences. So it is not with how mathematics does its business with which Husserl is concerned but rather that mathematics would have the audacity to expand its domain over the first and most genuine science, i.e. philosophy. It should be understood that this is a slightly more nuanced view than that taken by da Silva, for example, who seems to read Husserl as taking a more authoritative stance against the superficialisation of the sciences themselves. In da Silva's opinion the cross-pollination between domains that is characteristic of contemporary mathematics, and that is essential to scientific discovery, is a "liberality" which Husserl would have forbidden.¹⁵ This is very much related to his reading of *Crisis* that revolves around a presumed disdain for the degeneration of formal mathematics, from science, to a technique devoid meaning. But for Husserl formal mathematics hasn't degenerated into a technique, it has transformed into such of essential necessity.

In some sense the formalisation of mathematics plays a dichotomous rôle in the history of philosophy, the teleology of which Husserl is investigating. It is the moment of "discovery-concealment" which condemns reason but at the same time offers it the means of its own saviour.¹⁶ It is only through the philosopher's mathematisation of himself and God that the possibility of a radical inquiry into subjectivity is opened up. Radical phenomenology can begin only once philosophy has had its head chopped off. It is with Descartes that the idea of philosophy as a "universal mathematics" finds its primal foundation, but this is not to suggest that Descartes had this idea at hand, in full clarity or as conscious motive. It is only once seen through the lens of a historico-critical investigation, and in connection with Leibniz's *mathesis universalis*, that this idea first comes to light in any kind of maturity. And this idea was still progressing in Husserl's time, and only first reaching some kind of clarity, in the guise of the "lively research" into the mathematics of definite manifolds that was taking place.¹⁷ This shows not only how the full sense of a

¹⁵ da Silva, "Mathematics and the Crisis of Science," 355.

¹⁶ Husserl, *Crisis*, 53.

¹⁷ *Ibid.*, 74.

scientific discovery can make itself known over the course of its history, but at the same time it highlights the interplay that takes place between philosophy and mathematics in the revelation of their respective *teloi*. Husserl is of course not against the idea of a *mathesis universalis* as such, rather he devotes much of his attention to this very topic in *Formal and Transcendental Logic* (henceforth *FTL*), but he sees this infinite project as falling within the region of mathematics. As pure analysis, formal logic is largely – if not exclusively – the domain of formal mathematics and this means that it is well past time that the philosopher “hand over his temporary foster-child to their natural parents”.¹⁸ This relinquishment of control over the development of true theories is interpreted here as Husserl’s way of delineating his philosophical ambitions from those of the formal mathematician. A choice has been made here to read this as a somewhat strategic move on Husserl’s part, even if this was not his conscious intention. By giving up custody of formal logic, Husserl is able to open up a region of study that will belong solely to phenomenology, allowing it to be placed on par with the exact sciences without needing to share their objective theme. Read in this way it is not a crisis of mathematics that motivates Husserl but rather it is the aversion of a crisis of philosophy that he takes as his motive.

There is also, undeniably, the question of a crisis of psychology, and it is even worth noting that the original title of the lectures that make up the text of *Crisis* was “The Crisis of European Sciences and Psychology”.¹⁹ So phenomenology as described by Husserl in *Crisis*, and to an even greater extent in other works from his later period, has undeniable connections and similarities with psychology. Looking back to “Philosophy as Rigorous Science” however, a slightly different view is presented. There it is rather the differences between the two sciences that Husserl wants to emphasise when he says that phenomenology is to be unequivocally a science of consciousness but not one which is to be confused with the empirical studies of psychology. Both involve a thematisation of consciousness to be sure but they at the same time find themselves held firmly apart by a fundamental difference in “orientation”.²⁰ Obviously this paper’s very motivation relies on leaning towards the

¹⁸ Edmund Husserl, *Logical Investigations, Vol. 1*, ed. Dermot Moran, trans. J.N. Findlay (Oxon: Routledge, 2001), 159.

¹⁹ Husserl, *Crisis*, 3.

²⁰ Edmund Husserl, “Philosophy as Rigorous Science,” in *Phenomenology and the Crisis of Philosophy*, trans. Quentin Lauer (New York: Harper Torchbooks, 1965), 91.

image painted by the earlier Husserl, and even involves a widening of the distance between the two fields of study, but this will hopefully not be understood as a denial of the value of a phenomenological psychology or as a claim of having found the one “true” phenomenology. For the purposes of this study however there must be established a more mathematical interpretation of the transcendental-phenomenological project and likewise an attempt must be made to imbue Husserl’s words with a sense that is coherent with this paper’s contentions. This involves trying to show that what seems like a move by the later Husserl towards a more psychological phenomenology is in fact only possible because it is guided by an understanding of modern mathematics which at all times remains operative in his thought. What is at the very least being claimed then is that there are surely some mathematico-logical insights which lie concealed in the shadows of Husserl’s own genius. But this is not to suggest that a phenomenological mathematics could, or should, be responsible for all transcendental problems that face the phenomenologist. Nor is it the advocacy for a turning away from questions of subjectivity to purely objective concerns. In fact, perhaps it could be said that this separation of psychology and mathematics is merely a methodological necessity in what is ultimately an attempt to diffuse the border between the two.

§7 The question of a critique of scientific methodology

As alluded to earlier, according to Husserl the mathematical sciences have effectively succeeded in shrouding the life-world, the world of immediate experience, in a “garb of ideas”.²¹ But it is important to note that Husserl does not hope to explain away this web of objective scientific truths, but rather considers these idealisations to be “methodically necessary”.²² That is to say that while this critical situation within which Husserl finds European civilisation does indeed call for a critique, it is one of the genuine scientific nature of the positive sciences and most definitely not of their tried and true methodologies. Husserl himself is very careful to emphasise this point and is always very quick to hail the theoretical-technical accomplishments of the sciences as “a miracle”.²³ What this means is that Husserl does not aspire to intrude upon scientific method and, as phenomenologist, is himself barred from getting

²¹ Husserl, *Crisis*, 51.

²² *Ibid.*, 87.

²³ *Ibid.*, 66.

involved in the positings of working scientists. Rather what Husserl is lamenting is the loss of meaning that the ever-progressing technisation of the sciences leads to and – more importantly – the subsequent effect this has on philosophy.²⁴ For Husserl it is clearly the case that, since Galileo’s genius helped initiate the idealized nature upon which the sciences now rely, science has progressively developed into a mere technical thinking. It has lost any connection with the concrete life of experience upon which it relies for its very meaning, and it becomes of course phenomenology’s rôle to elucidate, and make genuine, its sense. Da Silva asserts however that by requiring of mathematics that it be instilled – by phenomenology – with sense, Husserl is in fact impeding upon its scientific progress.²⁵ So while Husserl claims to be simply critiquing the genuineness of the positive sciences, he in fact embroils himself in an indirect criticism of the way they carry out their business. Now the reason da Silva believes this to be the case is because of the aforementioned liberalities with which science needs to operate and which he believes Husserl to be opposed to. It can be agreed with da Silva that Husserl, despite his best efforts, does in fact get involved with scientific method, but there is perhaps a difference as regards the question of just how he manages to do so. Even more pressing however is whether any intrusion by philosophy into the scientific work of the exact sciences should in fact be designated as something to be avoided.

Both Husserl and da Silva would no doubt agree that it is undesirable for phenomenology to become involved in the methodologies of the mathematical sciences but here quite the opposite will be proposed. As part of the “division of labor” that Husserl draws up in *Logical Investigations* (henceforth *LI*) he makes a pointed note of surrendering the contested region of syllogistic logic to mathematicians and, in fact, scorns other philosophers for trying to get involved in a decidedly mathematical question.²⁶ The mood has changed somewhat by the time of *Crisis* however, now Husserl is taken aback by the mathematics community’s resistance to any extra-mathematical assistance and their labelling of genuine philosophical insight as merely “metaphysical”.²⁷ The sense that could possibly be extruded from these differing situations is that there is an obvious struggle at play

²⁴ *Ibid.*, 46.

²⁵ da Silva, “Mathematics and the Crisis of Science,” 350.

²⁶ Husserl, *LI*, §71.

²⁷ Husserl, *Crisis*, 57.

concerning the setting of clear boundaries between philosophical and mathematical work and that all attempts at this delineation seem to suffer from the difficulties inherent in communication between the two communities. If phenomenology is to truly critique mathematics, that is to say be involved in deciding the limits of its region, then it must in some way be involved with questions of methodology. How else can it hope to understand the limitations that formal mathematics experiences? As Ralph Krömer astutely points out, it is only possible for philosophy to revitalise science via “an interaction (transforming both science and philosophy)” and this requires philosophy to involve itself in the science contemporary to its time.²⁸ This means that not only would phenomenology be expected to involve itself in the critique of logical reason but that, going one step further than this, it must also open itself up to the possibility of being critiqued by mathematics. This would mean that where as Husserl’s strategy is interpreted here as one of surrendering a small piece of territory in order to gain unhindered access to an entire region, as this paper unfolds a more collaborative course will be suggested. Phenomenology must admittedly first encroach upon formal ontology but only in the interest of allowing mathematics access to the domain of transcendental science. This would involve of course mutual methodological involvement.

²⁸ Ralf Krömer, *Tool and Object: A History and Philosophy of Category Theory*, Science Networks. Historical Studies. 32 (Basel: Birkhäuser, 2007), 4.

II. FORMAL MATHEMATICS AND PHENOMENOLOGY

In the following chapter an attempt will be made to further demonstrate the delineation between mathematics and philosophy which is seen as resulting from the crisis of science as experienced by Husserl. This will involve an investigation of Husserl's concept of the definite manifold which – it will be claimed – acts as the perimeter between formal mathematics and phenomenology. The limitations that this concept imposes upon formal ontology, and the region that this subsequently opens up for phenomenology, will then hopefully be clarified. Alongside this there will also be an attempt at the circumscription of the naturalisation of phenomenology so that the concept of this project can be sharply separated from this paper's own. This will be done in the hope of establishing the reason why the contradictions affecting that initiative do not in any way endanger the arguments in the process of being advanced.

§8 Formal logic is formal ontology is formal mathematics

Husserl devotes a considerable amount of energy in *FTL* to delineating the concepts of apophantics and formal ontology before ultimately uniting them, once again, as a single science: formal logic. There are perhaps a multitude of reasons for Husserl's doing so but it could largely be seen as a necessary step towards removing any trace of psychologism from logic, understood in its most pregnant sense as the science of theory (or, considered correlatively, a theory of science). It is of course widely known that in his 'Prolegomena' Husserl is very much driven by this topic, but what becomes clear even in his later texts is that this question continues to remain an integral part of his motivation.²⁹ This is undoubtedly a question of demarcation for Husserl. In *FTL* he outlines how the expulsion of psychologism from logic makes clear that science and the critique of scientific reason are two very separate fields of study, formal mathematics being of course the science of science and phenomenology the science responsible for any critique.³⁰ In this way Husserl hopes to show that the two sciences, both eidetic, can carry on with their own tasks undisturbed by the other. Following on from this it is obviously the case that the formal-ontological studies enacted by formal mathematics have no concern for the fact that their formations are

²⁹ Husserl, *LI*, 9ff.

³⁰ Husserl, *FTL*, 67.

to be used in cognitional judgments.³¹ It is in this way that Husserl conceives of theoretical mathematics as being entirely impractical. Whereas the applied mathematician works away at mathematical problems with practical ends, the formal mathematician is merely interested in playing with thoughts so to speak, and this is of course very much related to the teleological picture, originating in the theoretical motives of Euclid, which Husserl is attempting to paint. But formal mathematics, regardless of whether its practitioners are aware of it or not, can in fact be said to be a formal ontology – it is essentially to be understood as a study of categorial formations. That is to say that, as ontology (and as formal theory of science), it is thematically directed towards objects and, taken together with the claim that “without exception, objects ‘exist’ only as objects of judgments”, it also becomes necessary to establish the relationship this science has with formal apophantics.³²

Apophantics, as explained by Husserl, is the branch of logic which finds its beginnings with the investigation of syntactical structures as carried out by Aristotle. It is the science which takes the identical judgment as its theme and, as such, which abstracts from any act of cognitional striving.³³ This is an important point because it demarcates formal-logical science from phenomenological science in that it means that formal apophantics, while concerned with judgment, is not truly concerned with the subjective act of judging. That is something that only transcendental phenomenology can take as theme. Inherent in formal logic is a “naïve presupposing of a world”, so even if formal apophantics and formal ontology are unified into one science, as Husserl sees them being, they are still ranked alongside the positive sciences.³⁴ This is because formal logic is concerned with an objectivity which finds its base in a modalisation of the actual world. So whereas formal apophantics and formal ontology are in some sense to be kept separate, and their ultimate unification is to be characterised as formal logic, all of these different names for one and the same science are at the same time essentially names for a study of possible worldly objects. This is due to the fact that to judge is to judge about objects and all objects, of necessity, presuppose a possible world. As a result of this, even formal logic could be called by the name formal ontology and this is always to be seen as the purview of

³¹ *Ibid.*, 109.

³² *Ibid.*, 120.

³³ *Ibid.*, §17.

³⁴ *Ibid.*, 225.

formal mathematics (and not, of course, a question which is genuinely philosophical). By enacting this investigation of the logical sciences, which are now to be understood as objective, Husserl has opened up for the possibility of a new science – transcendental logic – which is thoroughly subjective but at the same time not psychological, not in the positive science’s sense of the word at any rate. Now even though Husserl very quickly moves from his initially formal-ontological interests to the more fundamental question of a “genuine mundane ontology” which is to make up the theme of phenomenological investigations moving forward, the importance of formal logic should not be underestimated.³⁵ That is to say that even though Husserl manages to discover the subjective groundings of the objective sciences, formal ontology is for Husserl a “nomological science which deals with the ideal essence of science as such” and as such it is a science very much involved in both the construction and investigation of objectivity.³⁶ But the question remains as to whether the formal-ontological domain is to be one associated with phenomenology or mathematics. For Husserl, who had willingly surrendered custody of this philosophical foster-child, the answer is quite clear but hopefully this point of view will be somewhat problematised over the course of the following discussions.

§9 From the theory of manifolds to the concept of the definite manifold

The theory of manifolds is, for Husserl, nothing less than the theory of science itself. While it may belong, as task, to formal mathematics it is related to the elucidation of all possible forms that any scientific theory may take. As Husserl puts it, the mathematician may have once been solely concerned with number and quantity but, over the course of history, their domain has expanded to one of even greater importance.³⁷ So when Husserl talks of logic, or mathematics, or ontology he is referring to a theory of science, responsible for investigating the possible forms, or manifolds, that all deductive systems must adhere to. For Husserl then, a manifold, in the pregnant sense, is the form of an “infinite object-province” which can be unified under the exact laws of a nomological science.³⁸ A manifold defines, from a finite

³⁵ Ibid., 291.

³⁶ Husserl, *LI*, 152.

³⁷ Edmund Husserl, *Philosophy of Arithmetic: Psychological and Logical Investigations with Supplementary Texts from 1887-1901*, trans. Dallas Willard (Dordrecht: Springer Science+Business Media, 2003), 409.

³⁸ Husserl, *FTL*, 95.

number of axioms, an infinite province that can be theoretically explained. This means that a manifold can contain no truth which is not deducible from its axioms and, taking this to the extreme, it gives rise to the idea of the definite manifold: a complete system of axioms with no need, or possibility, of further explanatory laws. That is to say that even though the definite manifold may be infinite in regards to the truths it may reveal, as a theory it is complete in the sense that its axioms have exhausted all possibilities as related to its region of theoretical interest. The definite manifold can completely explain its scientific field, assuming of course there is a community of scientists on hand to work away at the infinite task of doing so. So the definite manifold is not just the form that a theory can take, but is rather a definitely deduced possible theory. This conception of the definite manifold is what lies at the very limits of formal mathematics in terms of being the ideal towards which it strives. It is, for Husserl, this concept with its hidden meaning-fundament, that is the very end towards which mathematics is steered. But Husserl actually doesn't go as far as to state this is an essential necessity defined by the very sense of mathematics, in fact the best Husserl can muster on the teleological rôle of the definite manifold is a "so it seems to me".³⁹ So does mathematics just seem to be guided by the concept of the definite manifold or is the idea of mathematics truly exhausted by a striving towards the systematic deduction of the whole science-form from a finite number of axioms?

The idea of the definite manifold is of course one which Husserl developed independently, having begun this work as early as the 'Prolegomena' and already having presented it in detailed form before the Mathematical Society of Göttingen in 1901 as his 'Double Lecture'.⁴⁰ But at the same time it is a concept which, as Husserl himself enthusiastically highlights, finds historico-critical support in the form of David Hilbert's work on completeness.⁴¹ As Mirja Hartimo points out, Husserl sees his own concept of definiteness as being very much related to the "Euclidean ideal", that is to say, a mathematical science driven by the exactitude of an exhaustive axiomatic system.⁴² Read with this in mind it becomes rather obvious that the mathematics developing at Göttingen, personified in part by Hilbert, helped Husserl to map out the teleological path of theoretical reason which he sees as having begun cleaving itself

³⁹ Ibid.

⁴⁰ Husserl, *LI*, §69; Husserl, *Philosophy of Arithmetic*, 409ff.

⁴¹ Husserl, *FTL*, 96.

⁴² Mirja Hartimo, "Husserl on Completeness, Definitely," *Synthese*, November 28, 2016, 17.

from philosophy at the time of Euclid and which he now finds continuing further upon this path of meaning transformation along with the development of the formal mathematics. Now, as Mirja Hartimo explains, there is far from unanimous agreement on the exact details of Husserl's conception of the definite manifold but in her view it is one which is purposefully both syntactic and semantic, with these dual aspects relating to the truth or falsity of statements and the uniqueness of a theory respectively.⁴³ That is to say that with the concept of the definite manifold Husserl needs to capture not only the way in which a theory must be deduced but also expressed. This is an important point because Hartimo here insists on finding a coherence between not only Husserl's concept of definiteness with the completeness theorems of Hilbert, but also with Husserl's own much wider philosophical project. So not only does the definite manifold, and formal mathematics' unflinching focus on complete axiomatisation, play an important historico-critical role for Husserl but it is also a central part of his own phenomenological work.

§10 Beyond the unity of the manifold

Now phenomenology, as science, also has a "system-form" that it must abide by, but the formal unity of its infinity of propositions is defined by the object of its study, not by a homogeneity of laws.⁴⁴ While it may be an eidetic science, phenomenology is descriptive and not explanatory: phenomenology does not have a deductive theory as its system-form. The fact that phenomenology has this essential difference from the nomological sciences is of central importance to the division of labour between mathematics and philosophy and to the whole of Husserl's project. The following point is not to be underestimated: phenomenology, as system, does not have the form of a mathematical manifold. This means that in order to understand the unity of phenomenology it is necessary to go "beyond the analytico-logical form".⁴⁵ But it is not just the system-form of phenomenology which escapes formal mathematics in this way, but also its region. That is to say that the infinite manifold of experience, which phenomenology takes as its theme, cannot be considered as definite. Now, to be sure, what the mathematician calls a manifold can be an infinite province but not infinite in the sense of the limitlessness of concrete experience. So when Husserl asks whether

⁴³ Ibid., 9.

⁴⁴ Husserl, *FTL*, 102.

⁴⁵ Ibid.

or not “the stream of consciousness” is “a genuine mathematical manifold,” this question can be promptly answered in the negative.⁴⁶ The idealisations with which formal mathematics busies itself are made definite by their limits – i.e. their finite number of axioms – and this places phenomenology’s region out of their reach. So where it can be said that both formal mathematics and phenomenology are eidetic sciences there is a difference in that they are analytic and synthetic respectively. Both admittedly deal with universalities but whereas formal ontology is interested in empty forms, phenomenology’s concerns are purely material. This is of course not to be confused with the material ontologies of the positive sciences, rather transcendental phenomenology is interested in the thematisation of the material *a priori*. Now the broadest, most universal, concept of formal mathematics is the “anything whatever” and, from Husserl’s point of view, it is important to note that this “anything whatever”, despite the fact that it is completely void of content, cannot be thought without the intentional constitution that is its correlate. So, following on from this, a most significant philosophical task becomes evident, one that is of an “essentially new” and which has a “strictly scientific style”.⁴⁷ This is the task of grasping the way in which every ontic *a priori* is related to the *a priori* constitution which necessarily precedes it. The ontic essential form, the *eidōs*, which at the highest level is called “category”, cannot be concretely possible without its constitutional essential form.⁴⁸ Every object, and that is to say even every category, must be formed in relation to an intentional process. And, as this intentional process is a correlate to the objectivities of formal mathematics, this philosophical task falls outside of its domain and naturally finds itself in the purview of transcendental phenomenology. For Husserl, phenomenology is the science which necessarily goes beyond the unity of the manifold to inquire back into its unification (its synthesis). This is something formal ontology cannot do.

⁴⁶ Edmund Husserl, *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy: First Book: General Introduction to a Pure Phenomenology*, trans. F. Kersten (The Hague: Martinus Nijhoff Publishers, 1982), 165.

⁴⁷ Husserl, *FTL*, 249.

⁴⁸ *Ibid.*, 248.

§11 Zermelo's paradox and the division of labour

Phenomenology's need of going beyond the concept of the mathematical manifold can also be felt in the absurdities which are occasionally reached by formal mathematics. Now Russell's paradox is not only well-known within the mathematical community but rather, more remarkably, has also been the focus of much attention within the lay community at large. In brief the paradox can be summarised by saying that if there were to be a set of all sets it would be required to include itself as one of its objects, while it at the same time cannot possibly do so. It is not insignificant to note that, thanks to the work of Ernst Zermelo, Husserl was in fact already aware of this antinomy at least a few months before Russell himself.⁴⁹ In her analysis of Husserl's undated manuscripts on set theory Claire Ortiz Hill points out that Husserl's view of Russell's paradox is that it merely originates from an unnoticed transformation of sense and as a result rests upon absurdity.⁵⁰ Said in another way, it results from a confusion of meaning that does not affect mathematical methodology but rather simply brings to light the need for conceptual clarification. Phenomenologically speaking, a set is a categorial formation and as such is formed by a performance of intentionality; a set is a collection, and a collection is necessarily the result of an act of collecting. This is what the skilled technician ignores when they are working away at perfecting their *techné* with no regards for ordinary meanings. It is precisely this transformation of sense that results in absurdity. But that is not to say that the mathematician is at fault here or that the mathematics is in crisis; the ongoing technical work within mathematics is still legitimate and necessary despite such philosophical difficulties. It is rather that this kind of philosophical paradox simply highlights the limits of objective science and the resulting need for phenomenology. A set comes to the mathematician as pre-given; it is the ultimate substrate with which they operate but the origin of which they are unable to inquire into. In Husserl's view, sciences operating with their concepts clouded by such paradoxes "are not sciences at all" and should be considered nothing more than "mere theoretical technique".⁵¹ That is to say that it is formal mathematics' inability to account for its own legitimacy that

⁴⁹ B Rang and W Thomas, "Zermelo's Discovery of the 'Russell Paradox,'" *Historia Mathematica* 8, no. 1 (February 1, 1981): 15–22.

⁵⁰ Claire Ortiz Hill, "Incomplete Symbols, Dependent Meanings, Paradox," in *The Road Not Taken: On Husserl's Philosophy of Logic and Mathematics* (London: College Publication, 2013), 250.

⁵¹ Husserl, *FTL*, 181.

means it must turn to phenomenology if it is to obtain any genuineness in its scientific endeavours. So not only can mathematics not account for consciousness or the infiniteness of lived experience: it can't even account for itself. The fact that mathematics comes up against the very limits of its region means that Husserl is able to enact a definitive delineation between philosophy and mathematics. Phenomenology, as synthetic *a priori* science, earns its place alongside the analytic *a priori* of mathematics. Or so Husserl's division of labour seems to suggest. What this hopefully helps to highlight is that much of the bedrock of Husserl's later philosophy is already being formed by the mathematical discussions taking place in Göttingen at the turn of the century, and that this contentious mathematical issue is still very much palpable in his work as carried out in *FTL* and *Crisis*.

§12 The absurdity of a naturalised phenomenology

If the previous sections have helped elucidate the delineation of philosophy by discussing the limitations of formal mathematics then it may now prove useful to examine an attempted encroachment upon the phenomenological domain. This act of intrusion is to be christened "the naturalisation of phenomenology". Jean-Michel Roy et al. describe the ongoing project of naturalisation, which has made itself known as a trend within phenomenological research, as one that aims at integrating phenomenology into "an explanatory framework".⁵² Now Roy et al. are not unaware of Husserl's opposition towards any attempts at naturalising phenomenology, but to talk of a hostility towards any project of transforming phenomenology into a mere specialization of the positive sciences is to miss the extent of the argument against any such attempts. It is important to note that for Husserl any naturalization of philosophy results in absurdity, that is to say that is not only undesirable but also completely countersensical.⁵³ Nowhere are the incoherent consequences of naturalistic philosophy better exemplified than in Hume's sensualism which, as a theory which attempts to show the very impossibility of theory, "is not merely wrong, but basically mistaken".⁵⁴ So the naturalisation of phenomenology is not only

⁵² Jean-Michel Roy et al., "Beyond the Gap: An Introduction to Naturalizing Phenomenology," in *Naturalizing Phenomenology: Issues in Contemporary Phenomenology and Cognitive Science*, ed. Jean-Michel Roy et al. (Stanford: Stanford University Press, 1999), 1f.

⁵³ Husserl, "Philosophy as Rigorous Science," 78.

⁵⁴ Husserl, *LI*, 75.

something to be opposed but is rather – of its very essence – self-refuting. In Roy et al.’s view however Husserl also had theoretical interests motivating his “anti-naturalism” which can be traced back to the limitations of the physico-mathematical science of his day and its failure to scientifically reconstruct the world of immediate experience.⁵⁵ Actually, here there cannot be complete disagreement with this claim as the limitations of the mathematics contemporary to Husserl’s time occupies an important rôle in this paper’s very own thesis. As there is undoubtedly some kinship between the project that Roy et al. support and the one advocated here it is perhaps necessary to elucidate the problems inherent in a naturalisation of phenomenology in order to further highlight the difference between the two.

In James Mensch’s presentation of the ongoing project of naturalizing phenomenology he provides a very handy example in which he illustrates a fundamental problem involved with the modelling of intentionality by mathematics with positive foundations. In demonstrating how it is possible to mathematically model the most simple of retentional processes he uses the following notation: “i, (i), ((i))...”.⁵⁶ What Mensch is trying to capture here is an originary impression being retained in iterative steps of retention (the retention of a retention of a retention...) and, while he may never explicitly declare it, what Mensch is also doing is modelling this process using the theory of sets. Now seeing as a set is equivalent to its members and two sets are considered equal if they share the same elements, then the attempt to mathematically model the retentional process which Mensch is outlining results in an obvious absurdity i.e. the model suggests that there is no difference between an originary experience and the memory of it. It is of course of essential necessity that a memory be a memory of an experience and not simply the experience itself. Now this is not to suggest that there is something necessarily wrong with presenting the model symbolically this way, it is rather with the actual mathematics – performed by Mensch – that there is a fundamental difficulty: the intentionality of transcendental subjectivity cannot be mathematised in this way. Retention is an intentional performance and as such it has its own essential structure. The problem with sets, like the ones being used by Mensch, is that they are pre-given as already formed and

⁵⁵ Roy et al., “Beyond the Gap: An Introduction to Naturalizing Phenomenology,” 40.

⁵⁶ James Mensch, “The Question of Naturalizing Phenomenology,” in *The 41st Annual Meeting of the Husserl Circle* (The 41st Annual Meeting of the Husserl Circle, New York, 2010), 65.

cannot capture the act of formation which characterises the intentional nature of the retentional process. Thanks to the axiom of extensionality these sets are defined by their elements but, as Ortiz Hill points out, even at the time of his early mathematical work, Husserl's was not affected by this "blight" of set theory.⁵⁷ Now Mensch tries to get around this restriction by equating consciousness with the concept of a function.⁵⁸ This allows him then to manipulate these sets with a few simple lines of computer code thus emulating the way in which consciousness works. This indicates how the naturalisation of phenomenology inherently takes previously formed sets and transforms them, via functions, into other sets. However, the problem is that transcendental consciousness is not just concerned with predicating about given categorial formations – constitution cannot begin with an object that is already formed by judgment but must be traced back to the "immediate cognitions" that are pre-categorial.⁵⁹ So what is exemplified by Mensch's work is that the naturalisation of phenomenology is restricted to the domain of the formal logician who is unable to enquire back into the genealogy of the ultimate substrates with which they are operating syntactically. Mensch claims to have precluded any objections to his method but this is only because before naturalising consciousness he is already operating with a naturalised conception of transcendental subjectivity. Restricted by the limitations of formal mathematics he cannot get past a cognitional consciousness to the intentionality of transcendental subjectivity. This can also be seen in the work of Null and Simons who, unlike the majority of proponents for a naturalisation of phenomenology actively aim at including the transcendental aspects of phenomenology in their mathematical work.⁶⁰ So, very much like the current paper is advocating, Null and Simons try to mathematise transcendental subjectivity in a way that cannot be construed as a neutralisation of its capacities. But as they themselves concede, their reliance on class-set theory model does not – and cannot – succeed in capturing the full extent of the theory they are espousing.⁶¹ So while Null and Simons

⁵⁷ Claire Ortiz Hill, "Husserl on Axiomatization and Arithmetic," in *The Road Not Taken: On Husserl's Philosophy of Logic and Mathematics* (London: College Publication, 2013), 110.

⁵⁸ Mensch, "The Question of Naturalizing Phenomenology," 69.

⁵⁹ Edmund Husserl, *Experience and Judgment: Investigations in a Genealogy of Logic*, trans. James S. Churchill and Karl Ameriks (London: Routledge & Kegan Paul, 1973), 24f.

⁶⁰ Null and Simons, "Manifolds, Concepts and Moment Abstracta."

⁶¹ *Ibid.*, 444.

also appreciate the importance of an investigation relating to the possibility of transcendental mathematics they are inevitably hindered by their formal-mathematical approach. For Mensch, Null and Simons, and Roy et al. it becomes obvious that the project of naturalising phenomenology is to be achieved by way of a mathematisation of phenomenology – with the proviso of course that this is carried out within the domain of formal mathematics. This paper also, to be sure, argues for the mathematisation of phenomenology but this is to be possible only after carrying out the phenomenologisation of mathematics – the aim is to leave the limitations of formal mathematics behind. Despite their obvious similarities then, it is argued that this fundamental difference between the two projects makes them of two opposing species. Before moving forward it should finally be remarked that this proposed absurdity of naturalised phenomenology is not to be understood in the sense of a ridiculousness of some sort. It is self-refuting only by the fact that it objectifies transcendental subjectivity in its attempts to achieve a mathematisation of phenomenology. This does not mean that it is useless or a failure in some way – it has shown, and will continue to show, its relevance to cognitive science. The point is rather that by excluding transcendental subjectivity from its studies, intentionally or of necessity, it sacrifices an invaluable aspect of Husserlian phenomenology – this is one of the central opinions, and in part the motivation, of this paper.

III. THE PHENOMENOLOGY OF LOGICAL REASON

By not penetrating to the level of constitutive intentionality, which remains hidden from the subject in their cognitional striving, the traditional logician fails to account for their own subjectivity in their investigation of knowledge-formations.⁶² With their epistemo-practical interests the logician aims at discovering how rational judgments are produced, but in their naïve attitude cannot get a view of their own rational striving, that is to say that the logician unreflectively aims at judging rationally about rational judgment. The logician takes the legitimacy of logical reason as presupposition, problem and conclusion. The critique of this logical reason becomes Husserl's theme in the second half of *FTL* but here, in chapter three, an attempt will be made to extend this investigation somewhat by preparing Husserl's findings for the zigzagging nature of the ongoing study. Said in another way, the following is an attempt to elucidate the limits and limitations of Husserl's critique by submitting it to a critique of its own.

§13 The double-sidedness of sense and the world of formal analysis

For Husserl the possibility of a judgment having sense depends not only upon its syntactical form but also its syntactical stuffs. Using the example of the proposition “[t]his color plus one makes three” he sets about clarifying the “double sense” inherent in the concept of sense as related to judgments.⁶³ This proposition immediately presents itself as nonsense in that, while it can obviously be uttered, it can never be proffered as a possible judgment. The material incoherence of the judgment-contents “color” and “one” means that, due to its senselessness, it is impossible for the proposition to be judged as either true or false. Considered purely formally however, as it would be by the formal-logician, this statement does at the same time have another kind of sense because it is logically coherent and, while it may not be possible to bring this sense to the clarity of intuition, it is at the same time non-contradictory. With this Husserl not only hopes to highlight the double-sidedness of the concept of sense but also the limitations of the formal-logical attitude in fully elucidating sense in all its aspects. While not making the possibility of judgments

⁶² Husserl, *FTL*, 40.

⁶³ *Ibid.*, 216.

thematic, it becomes abundantly clear that the whole of formal logic is built upon the proposition of a harmonious unity of experience. It relies upon “material homogeneity among its cores” and is related, of necessity, to “a unitary material province”.⁶⁴ That is to say that while formal logic cannot have anything to say about judgment-content, its very possibility rests upon material consistency and, said in yet another way: in order for analysis to bring the correctness of judgments to evidence it presupposes a possibility of judging that it can never prove. Because of this, formal analysis cannot investigate the conditions of its own possibility. So here Husserl sets about clarifying the intentional genesis of judgments as such and has at the same time highlighted the fact that there is a distinct need for another kind of logic, one that can explicate the very possibility of formal logic. Now formal logic, as apophantics, managed to hold itself to the analytic *a priori*, but due to its mediate connection to formal ontology grounded itself – unwittingly – upon the world as experienced in actuality. This is because formal ontology, with its thematisation of possible objects, relies inherently on the modalisation of an actual existing world. In this way formal logic in its fullest sense, that is to say as the unification of apophantics and formal ontology, relies upon the idea of a world as given beforehand. Any attempts to clarify the fundamental concepts, and their subjective aspects, only ever reached to mundane psychological investigations which, in the same way as formal logic, had an already-given world obscured from its thematic gaze.⁶⁵ In order to radically ground logic, and consequently the totality of objective science, the real world must be called into question. This is not to say that the actuality of the world must be doubted, but rather that the question of the full being-sense of the world must be posed. Formal logic inherently relies on this sense of the world as one of its fundamental concepts and, as such, this sense must be clarified if science as a whole is to be understood.⁶⁶ If logic is to be a genuine science then the meaning of the world must first be explicated.

§14 The transcendental-phenomenological reduction

In order to approach the phenomenological world-problem, it becomes necessary to perform the fundamental phenomenological method known as the reduction. This transcendental-phenomenological reduction can be understood as the laying bare of

⁶⁴ Ibid., 221.

⁶⁵ Ibid., 224f.

⁶⁶ Ibid., 229.

“concrete transcendental being” and the opening up of “the way to constitutional problems”.⁶⁷ Put in another way, the reduction is a matter of taking the region of transcendental subjectivity as scientific motive – as “region of theoretical discovery”.⁶⁸ Now while this motive has – from the very beginning of phenomenology – been definite, the full sense of this reduction to transcendental consciousness cannot be attained without a consideration of its various motivations. In his seminal work “The Three Ways to the Transcendental Phenomenological Reduction in the Philosophy of Edmund Husserl” (henceforth “The Three Ways”), Iso Kern elucidates three such motivations or “ways in”: the Cartesian way, the way through intentional psychology, and the way through the critique of the positive sciences and ontology.⁶⁹ Now, in Kern’s view, the first two ways are to be considered failures, aborted by Husserl himself, while the critique of science and ontology alone motivates the thematisation of transcendental subjectivity. And as “The Three Ways” is a historical study, Kern even peppers the text with philological evidence of Husserl’s own denials of the first two ways. But as was stated at the outset of this paper, if a historico-critical investigation is to be carried out, not even the words of Husserl himself can be taken *prima facie*. With that in mind, it is contended that it should not be understood that the Cartesian way and the way through psychology were both attempts at realising the full sense of the transcendental reduction that unfortunately ended in disappointment. Rather the ultimate sense of the transcendental reduction can only be fulfilled, through the critique of ontology, if this is understood to be a fulfilment that is, of necessity, by way of the preceding intentional disappointments. It is only together with an understanding of the inadequacy of Cartesian way, as a reduction to the indubitability of consciousness, and the psychological way, as reduction to the purely psychical, that the way through ontology can be truly understood. So when Kern remarks that Husserl never managed to clearly delineate between the three ways to the reduction this should be understood not as accident, but rather as something that touches upon the very essence of the

⁶⁷ Ibid., 268f.

⁶⁸ Edmund Husserl, *The Basic Problems of Phenomenology: From the Lectures, Winter Semester, 1910-1911*, trans. Ingo Farin and James G. Hart (Dordrecht: Springer, 2006), 73.

⁶⁹ Iso Kern, “The Three Ways to the Transcendental Phenomenological Reduction in the Philosophy of Edmund Husserl,” in *Husserl: Expositions and Appraisals*, ed. Frederick A. Elliston and Peter McCormick (Notre Dame: University of Notre Dame Press, 1977), 126.

transcendental reduction itself.⁷⁰ The Cartesian way purifies consciousness of nature, the psychological way purifies the soul (or psyche) of its intentional contents, and finally, the critique of ontology purifies transcendental subjectivity of any categorial form. So when Husserl, in *Ideas*, describes the phenomenological method as a "step-by-step reduction" the only way the sense of these words can be interpreted, *now*, is precisely in this manner.⁷¹ That is to say that rather than being individual "ways in" to the reduction, these three motivations are here interpreted as in fact three steps along the way to the legitimation of the transcendental-phenomenological reduction as genuine – presuppositionless – method. The reduction is in other words a reduction of a reduction. Again quoting from *Ideas*, it is thus claimed that this is the full sense with which Husserl's words should be imbued when he states that *multiple* reductions will collectively be referred to as "*the* phenomenological reduction".⁷² This is not to say that Husserl had before himself – in full clarity – this sense already at the stage when he wrote *Ideas* but rather that, as a result of zigzagging back and forth, the full sense of the transcendental reduction can be elucidated and understood in a way that Husserl never could – at least not at the stage of *Ideas* in any case. So when *the* transcendental-phenomenological reduction is spoken of it will simply be in reference to the thematisation of transcendental subjectivity and, while for the specific task at hand an understanding of the critique of the objective sciences may be the most pressing, there will at all times be an allusion to the full sense of the reduction as seen through its various motivations – "failed" or otherwise.

§15 Parenthesising *mathesis universalis*

Approaching the thematisation of transcendental subjectivity as described is to be carried it in various steps, one of which is the aforementioned critique of ontology. This amounts to a transcendental *epochē* as concerns all ontologies pregiven by the objective sciences: the material ontologies, such as biology and chemistry, and all the formal ontologies as encompassed by formal mathematics. Luckily enough this does not mean that there is a need to suspend belief in the various ontologies one by one – as this would most likely be an endless task. Rather, the transcendental *epochē* is able

⁷⁰ Ibid.

⁷¹ Husserl, *Ideas*, 66.

⁷² Ibid.

to disengage all ontic validities in “one blow” so to speak.⁷³ Formal ontology, as has hopefully established by now, deals with the “anything whatever” and as such “embraces all spheres of being and objects and, correlatively, all spheres of cognition”.⁷⁴ So by parenthesising formal mathematics the ability to suspend belief in objective science in its entirety arises. Now Husserl admits, in *Ideas*, that at first the possibility of parenthesizing formal logic in this way may seem somewhat questionable.⁷⁵ Even phenomenology has its region of objects and as such seems to be at the beck and call of *mathesis universalis* with its authority over the “anything whatever”. But what saves phenomenology from formal-ontological servitude, according to Husserl, is the presupposition that it restricts itself to descriptive analysis of intuition. This means that while phenomenology deals with concepts and judgements it does not do so in a constructive manner. “To avail ourselves of nothing but what we can make essentially evident by observing consciousness itself in its pure immanence,” this is the norm of phenomenology.⁷⁶ As Husserl states in ‘Appendix IV’ of *The Basic Problems of Phenomenology*: the phenomenologist does not judge about positive objectivities, but they do in fact judge.⁷⁷ So it is not a positive – or natural – judging which is performed by the phenomenologist but it is a pure form of judging, one directed solely towards subjectivity. Husserl also admits that there are certain logical axioms, the principle of non-contradiction for example, that are available to the phenomenologist even after the performance of the reduction, and Husserl’s prevalent use of the concept of absurdity to abrogate countersensical arguments only testifies to this fact.⁷⁸

Now Kern proclaims any philosophy founded upon positive science to be “nonsense”.⁷⁹ But actually this needs to be refined somewhat in line with the brief discussion on material incoherence earlier. If an objective philosophy was merely nonsense then it would not be possible to pass any judgement on it – it would be neither true nor false – so it is important to clarify that the idea of an objective philosophy is, more correctly, absurd. A philosophy founded upon the positive

⁷³ Husserl, *Crisis*, 150.

⁷⁴ Husserl, *FTL*, 291.

⁷⁵ Husserl, *Ideas*, §58.

⁷⁶ *Ibid.*, 136.

⁷⁷ Husserl, *Basic Problems*, 113.

⁷⁸ Husserl, *Ideas*, 136.

⁷⁹ Kern, “The Three Ways to the Transcendental Phenomenological Reduction in the Philosophy of Edmund Husserl,” 146.

sciences is not senseless, it is clear what the idea means, but rather it is countersensical – it is materially contradictory, i.e. necessarily false. Now, this point was already taken up in line with the discussions on the naturalisation of phenomenology but it becomes even more vital here as it becomes not just a question of a current trend within phenomenological research but one of the genuineness of science and its relationship to the phenomenological method. If the critique of ontology is to open the way in to the transcendental reduction then naturalising phenomenology would only serve to close off this entrance. This only helps to clarify the absurdity of a naturalised phenomenology as it highlights that the resulting positive philosophy would have no access to transcendental subjectivity and as such would lose any right to the name of genuine science. But the points mentioned earlier, those concerning phenomenological judgment and the necessity of logical axioms prior to the *epochē*, at the very least help to open for the idea of a mathematics which is immune to the phenomenological reduction. Even if, by way of transcendental *epochē*, formal logic truly falls to the wayside there are still some aspects of logic at play in Husserl's phenomenology that at the very least highlight a tension between the concepts of material and formal as he conceives them.

§16 Uncovering the soil of the objective sciences

So the phenomenologist must carry out an *epochē* in regards to the objective sciences, and this is achieved by parenthesising *mathesis universalis*. Hopefully it has been made clear that this does not mean imagining, in some form of eidetic variation, a world without science, but rather merely concerns abstaining from theoretical interests and scientific cognitions.⁸⁰ Scientists and their sciences do not disappear after the *epochē*, the phenomenologist just no longer partakes in their interests or positings. What this method reveals then is that the objective sciences, as theoretical praxis, presuppose the very world which they attempt to “predicatively interpret”.⁸¹ Positive science has set itself the infinite task of transforming the transiency and imperfect knowledge of the pre-scientific world into a perfect knowledge of a constant, determined world.⁸² It is by objective science that the pre-predicative becomes predicative and the indeterminate becomes determined. Science is a practical

⁸⁰ Husserl, *Crisis*, 135.

⁸¹ *Ibid.*, 111.

⁸² *Ibid.*, 110f.

accomplishment aimed at spiritual structures which are of the theoretical sort. Just like all other forms of praxis it is directed towards, and grounded upon, the pre-scientific world in which it also takes its place.⁸³ The physically existing sources of verification (measuring scales for example) are taken as something valid and as such play the part of premise in the act of scientific deduction as premise.⁸⁴ So in their attempts to overcome the subjective-relative nature of pre-scientific knowledge the scientist in fact takes as universal grounds for their objectivity the ontic validity of that which exists in the life-world. Questions then arise concerning the relation of pre-scientific and scientific objects to subjectivity and its ontic validities, but these questions naturally fall outside of the domain of objective science.⁸⁵ They are not questions that can be answered, or even asked, by a *technê* which takes as its concern objective truths.

By way of the universal *epochē* the phenomenologist rises above all interests which drive human praxis, and does not even have any sort of conception of the results that this new science may produce.⁸⁶ The phenomenologist at this point is no longer concerned with clarifying the meaning of the positive sciences, the motivation which first lead to the unveiling of the life-world in this way. For Husserl, regardless of how pressing the unsolved questions regarding objectivity may be, they must be set aside so that the life-world can be made thematic.⁸⁷ But David Carr, the translator responsible for rendering *Crisis* into English, points out that despite its fertility and insightfulness there is unequivocally some confusion with Husserl's conception of the life-world that is of fundamental importance to the phenomenological project as a whole.⁸⁸ The problem is that under the term "life-world" Husserl seems to include two separate phenomena: on the one hand the idea of a pre-predicative world of experience, and on the other a cultural world which is pre-scientific but not devoid of lower-level categorial formations. For Husserl it is by virtue of objective documentation that the logical formations of science, and by extension mathematics, can be returned to and reiterated by anyone. That is to say that it is the fact that

⁸³ Ibid., 118.

⁸⁴ Ibid., 126.

⁸⁵ Ibid., 111.

⁸⁶ Ibid., 156.

⁸⁷ Ibid., 133.

⁸⁸ David Carr, "Husserl's Problematic Concept of the Life-World," in *Husserl: Expositions and Appraisals*, ed. Frederick A. Elliston and Peter McCormick (Notre Dame: University of Notre Dame Press, 1977), 202–12.

anyone can pick up a mathematics textbook at anytime and, by a simple repetition of certain steps, enact a rediscovery of the mathematical truths embodied within, that makes the signification unities of mathematics the “common property of mankind”.⁸⁹ As Carr astutely points out, the theoretical world of objective science seems first to be grounded upon the cultural world – a world of books, language and categorial formations – before mediately finding its grounds in the nourishing soils of the world of immediate, perceptual experience.⁹⁰ The life-world is, and remains, untouched by science. But this does not mean that the common world of everyday experience is not in some sense enriched by science; the world indeed expands, so to speak, for every new scientific object which is discovered. As just one of many forms of human praxis objective science (along with its scientists, theories and discoveries) takes place within the life-world but it cannot alter the world’s general structure. The world may be viewed differently in light of scientific advancement, but the style of the world – of essential necessity – remains the same.

§17 The universal problem of intentionality

Despite the monumental unveiling of the life-world – whichever form it may take – Husserl does not allow himself to pause here long in his journey of discovery. Just as the problems of scientific objectivity needed to make way for the questions concerning concrete experience, the life-world itself must move aside so that intentionality can take centre stage as the preeminent theme of phenomenology. In *Ideas* Husserl will refer to intentionality as “a comprehensive name for all-inclusive phenomenological structures”, and this albeit rather vague definition will help in avoiding some obscurity.⁹¹ One common misconception is that with the term intentionality – understood as a “consciousness of something” – Husserl is in fact referring to the ability of active consciousness to shift its regard.⁹² But, if the attention of the cogito is drawn towards something, subsequently making a specific object thematic, this is to be understood simply as a particular modality of intentionality. That is to say: intention is not attention. The confusion arises here by interpreting Husserl’s use of the word “consciousness”, not in its most pregnant sense, which

⁸⁹ Husserl, *FTL*, 27.

⁹⁰ Carr, “Husserl’s Problematic Concept of the Life-World,” 211.

⁹¹ Husserl, *Ideas*, 199.

⁹² *Ibid.*, 201.

includes both active and passive consciousness, but rather purely in the sense of consciousness in its mode of actualising. But in order for an object to assert an allure on the cogito it must first “appear” as part of a horizon of potential perceptions and acts. This means that while the ego is at all times surrounded by a “potential field of perception”, and the objects within this horizon are “intended to”, they are not necessarily thematised.⁹³ So phenomenology, with its theme of intentionality, extends beyond a study of acts of egological cognition. This hopefully helps highlight that the logic which Husserl is most interested in is not one which is simply restricted to the syntax or semantics of categorial objectification. Not all objects within a Husserlian phenomenology are objects formed by predicative judgments. The objective logic of the sciences may be the first but it is not, for Husserl, the final. There is another “formal logic” – and here Husserl himself places the phrase within inverted commas – one which does not presuppose a possible world, but rather that investigates world-possibility.⁹⁴ This can be understood as a transcendental logic, which is not just restricted to the genealogical investigation of logical reason, or simply to the realm of categorial formations, but one which would be an “ultimate theory of science” in that, by turning (inwards) towards the transcendental, it would be able to account for not just science but also the world.⁹⁵ This point is brought up here as it raises relevant questions as regards the possibility of a mathematical science to account for objectivities that are not objective in the positivistic sense of the word. Restricted to a formal ontology it is necessary to agree that there is no possibility of accounting for pre-predicative objects but the question remains whether a transcendental ontology, which was at the same time mathematical, could extend its region to include the themes taken up by phenomenology.

⁹³ Ibid., 200.

⁹⁴ Husserl, *Crisis*, 271.

⁹⁵ Husserl, *FTL*, 16.

IV. CATEGORY THEORY AS TRANSCENDENTAL SCIENCE

It is essential to descriptive sciences that they are inexact; *ipso facto* mathematics is, as non-descriptive, an exact science.⁹⁶ Mathematics, with its ideal of exactness, has nothing to say about the vagueness of the region belonging to phenomenology. Or that is anyway the conclusion that would remain if Husserl was to be accepted at his word. But there is a sleight of hand – or at the very least a poorly-hidden presupposition – here and that is the assumption that all forms of mathematics are essentially ideal. Performing a transcendental *epochē* upon formal mathematics has been shown to be a necessary step if transcendental subjectivity is to be taken as a genuine theme, and in suspending belief in the objective sciences the phenomenologist simultaneously denies themselves the philosophical use of their technologies. This means that formal mathematics is no longer accessible to any further phenomenological investigations. But what if there is a mathematics that is not a performance of an idealisation? What if there is a mathematics that goes beyond the limitations of the mathematics that was pre-given for Husserl? This chapter will be devoted to this very topic.

§18 What is category theory?

The mathematical concept “category” was first unveiled by Samuel Eilenberg and Saunders Mac Lane in their 1945 article “General Theory of Natural Equivalences”.⁹⁷ But as Jean-Pierre Marquis explains Eilenberg and Mac Lane really only elucidated the idea of a category in order to explain the closely related concept of functor and, even more specifically, the notion of natural transformation with which they were more importantly concerned.⁹⁸ As category theory progressed, and as it started to actually become a theory and not just the definition of a handful of terms, the concept

⁹⁶ Husserl, *Ideas*, 166 [138].

⁹⁷ Samuel Eilenberg and Saunders Mac Lane, “General Theory of Natural Equivalences,” *Transactions of the American Mathematical Society* 58, no. 2 (September 1945): 231–94.

⁹⁸ Jean-Pierre Marquis, “What Is Category Theory?,” in *What Is Category Theory?*, ed. Giandomenico Sica, *Advanced Studies in Mathematics and Logic* 3 (Monza, Italy: Polimetrica, 2006), 229.

of category became more and more important for mathematics. A category is made up of two objects and an arrow describing the relationship between the two. Somewhat confusingly, dependent upon just what the objects are, an arrow goes by various different names: between general objects an arrow is called a “morphism”, between objects that are sets it is known as a “map”, between objects that are categories it’s called a “functor” and between functors, that is as the representation of a relationship between relationships, it is what is known as a “natural transformation”. The idea of a natural transformation stems from the fact that if there are various ways of transforming one category into another then there must be ways in which these transformations can also in turn be themselves transformed. In this way the arrows between categories become themselves objects for morphism. And for F. William Lawvere it is just a matter of intuition to say that these arrows, or natural transformations, themselves constitute a category.⁹⁹ In Lawvere’s way of describing the state of affairs, category theory really began when the morphisms between mathematical objects were viewed as a new kind of mathematical object and when, subsequently, this new object was recognised as belonging to a species which was itself independent of the objects with which it began.¹⁰⁰ Morphisms, while they may be mathematical objects, are not mere elements of a set. So while there are objects and arrows making up a category, arrows can themselves be objects and these objects can in turn be involved in categories of their own. While trying to come to terms with the difficult question of exactly what it is that category theory could be said to be, Marquis suggests that answering this question requires an understanding of its possible uses, and so with some basic concepts in hand it is perhaps necessary to explore a little further in order to elucidate just what category theory might be.¹⁰¹

As Robert Goldblatt explains, the advent of category theory brought about a considerable shift in view within the field of mathematics.¹⁰² Whereas modern mathematics in the earlier part of the 20th century (i.e. in Husserl’s time) was

⁹⁹ F. William Lawvere, “The Category of Categories as a Foundation for Mathematics,” in *Proceedings of the Conference on Categorical Algebra in La Jolla 1965*, ed. Samuel Eilenberg et al. (Berlin: Springer-Verlag, 1966), 9.

¹⁰⁰ F. William Lawvere, “Adjointness in Foundations,” *Dialectica* 23, no. 3–4 (December 1969): 281.

¹⁰¹ Marquis, “What Is Category Theory?,” 222.

¹⁰² Robert Goldblatt, *Topoi: The Categorical Analysis of Logic*, Revised, vol. 98, *Studies in Logic and the Foundations of Mathematics* (North-Holland: Elsevier Science Publishers, 1984), §1.3.

consumed by describing the structure of mathematics in terms of sets, contemporary mathematics brought with it the possibility of conceiving of mathematical objects as something other than a mere collection of elements. Goldblatt helps in elucidating just what category theory is all about by explaining that one way that a category can be described is as being a “universe of mathematical discourse”.¹⁰³ Seen in this way a functor can be seen as a transformation which turns one universe into another and its inverse, an adjoint functor, could be seen as an attempt to trace the genealogy of an object back to its origins. Described in this way, category theory can be seen as the study of the way in which mathematical objects take form, and this science starts from the clarification of the concept of morphism. So it is not the precise structures of mathematics, but rather the “mutability” of these structures, which category theory takes as its theme.¹⁰⁴ Category theory leaves the binary relationship of membership behind and shifts attention to the ways in which objects are constituted. Now the idea of the transformation of mathematical objects does not first arise with category theory of course, the concept of homomorphism comes to it from algebraic topology and set-theoretical mathematics more generally already has its isomorphisms – that is to say that the study of mappings was already pre-given even in Husserl’s time.¹⁰⁵ But whereas homomorphism is a structure-preserving map between two sets, which are both elements of the same manifold, a morphism is the generalisation of this concept to apply to any object whatsoever.

One aspect of category theory not to be underestimated is the commutative diagram. These are the diagrams which allow for the tracing of paths between potentially distant mathematical universes and which are said to be commutative by the fact that they demonstrate that regardless of which path is followed the resulting object remains, or becomes, the same. Marquis retraces some of the history of category theory and highlights how, in combination with its key concepts, commutative diagrams help to make highly abstract areas of mathematics “intelligible”.¹⁰⁶ In fact in his description of just what it is to give a category, Lawvere states that it is simply a matter of giving meaning to “the word morphism and the commutativity of diagrams like”:

¹⁰³ Ibid., 98:1.

¹⁰⁴ F. William Lawvere, “Taking Categories Seriously,” *Revista Colombiana de Matematicas* 20 (1986): 148.

¹⁰⁵ Krömer, *Tool and Object*, 40.

¹⁰⁶ Marquis, “What Is Category Theory?,” 234.

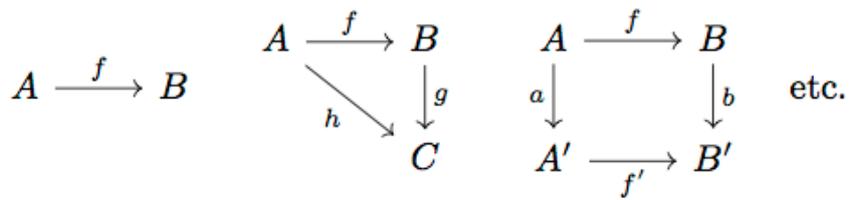


Figure 1. Some commutative diagrams¹⁰⁷

So if it is indeed as Marquis believes, that to say what category theory is to say what it can be used for, then it is argued that what Lawvere sees the concept of category as doing is instilling morphisms and the commutativity of diagrams with meaning by making them useful – i.e. by giving them a practical purpose. Armed with these tools the contemporary mathematician can undertake heretofore-unimaginable tasks. A functor, as Marquis explains, is a “mathematical representation of a conceptual transformation” that is ambivalent to the type of object it is operating on, and an adjoint functor on the other hand could be described as the attempted (i.e. approximate) transformation of an object back into the object it originated from.¹⁰⁸ So the task undertaken by category theory, that which gives it its meaning, is one of investigating the intentional performances of mathematics. By way of “diagram chasing” and the concept of adjointness, the category theorist can investigate not only the formation but also the de-formation of mathematical universes. In Marquis’ opinion it was along with the introduction of adjoint functors that category theory gained some autonomy as it gave category theory some reflexivity, in terms of allowing it to account for its own concepts, and it allowed the likes of Lawvere to claim that sets could be defined in terms of categories instead of needing sets to define categories.¹⁰⁹ It could be said that it allowed category theory to escape the limitations of formal mathematics and transformed mathematics into a thoroughly conceptual affair. With the advent of category theory mathematics is no longer mathematics, as Husserl envisaged it, because now the once purely formal science seems equipped to enact its own self-critique.

¹⁰⁷ Lawvere, “Adjointness in Foundations,” 283.

¹⁰⁸ Jean-Pierre Marquis, *From a Geometrical Point of View: A Study of the History and Philosophy of Category Theory*, Logic, Epistemology, and the Unity of Science 14 (Dordrecht: Springer, 2009), 152.

¹⁰⁹ Marquis, “What Is Category Theory?,” 242.

§19 Subjective and objective logic

If category theory is to represent a contemporary attempt at the critique of logical reason, as is being suggested here, then it must be shown to concern itself with subjective logic in the Husserlian sense. The first obvious stumbling block in the pursuit of the paper's motivation is the fact that Lawvere very clearly refers to category theory specifically as the pursuit of "objective logic".¹¹⁰ As Lawvere puts it, category theory arose from the conceptualisation of the study of space and quantity, that is to say from needs felt within geometry for conceptual clarification. But it is by a shift of focus from merely geometrical objects, and a widening of its region to include "the concepts and their interactions" which arise from the study of "any serious object of study" in general, that Lawvere steers category theory towards his conception of objective logic.¹¹¹ So, seen in this light, category theory could be described as a theory of theories or as a theory of science. Category theory can thematise, translate and encompass much of symbolic logic and formal mathematics within its own region. It is contended here that this can be interpreted as meaning that, in precisely the same way as Husserl, Lawvere envisages apophantics and formal ontology as equivalent and inseparable parts of a single science.¹¹² Logic is ontology is mathematics. Put in another way, category theory sets in action the actualisation of a large part of the claims made by Husserl in *FTL*. An objective logic, for Lawvere, is aware of its necessary correlation to ontology so with this in mind a direct correlation between category theory and *mathesis universalis*, as conceived by Husserl, seems to have been elucidated. Category theory seems to be equivalent to the theory of manifolds and, taken one step further, could possibly be considered as the full realisation of this mathematical project as outlined by Husserl.

If category theory has these epistemological aims, or perhaps more accurately faculties, surely this means that the argument that it represents a transcendental turn within mathematics comes to a stand still. Even if it could be said to be equivalent to the theory of the manifolds this was, for Husserl, nothing more than the goal of a formal mathematics. It may help to mention that Lawvere's view of matters is that the

¹¹⁰ F. William Lawvere, "Tools for the Advancement of Objective Logic: Closed Categories and Toposes," in *The Logical Foundations of Cognitive Science*, ed. John Macnamara and Gonzalo E. Reyes, vol. 4, Vancouver Studies in Cognitive Science (Oxford: Oxford University Press, 1994), 43–56.

¹¹¹ *Ibid.*, 43.

¹¹² Husserl, *FTL*, 111.

mathematical theory of categories is an essential element in the successful development of the “science of knowing”.¹¹³ What is of interest, for the task at hand, is to take note of the fact that Lawvere is directed towards a science which would take “knowing” – not “knowledge” – as its subject. That is to say that what piques any interest in category theory is that it allows for the thematic focus upon cognitional acts and not just upon pre-formed objects of cognition. It is the mutability of mathematical objects, not immutable structures, with which category theory is concerned. Importantly, category theory treats the mathematical universe as a construction of human activity so even if it is to act as a form of objective logic there is an obvious need for this to be done by way of a thematic focus upon mathematical subjectivity. When regarded in this light it could be argued that category theory, with its thematisation of morphism, turns its gaze from mathematical forms towards the idealising acts carried out by a community of mathematicians. Now, it has been shown by Husserl that formal logic has as its theme ideal formations and it is only by apprehending this fact that transcendental questions can first be posed.¹¹⁴ So following on from this what is obviously being suggested is the possibility that category theory has an important rôle to play in the realisation of any transcendental critique of logical reason. The claim then is quite a bold one, and that is this: category theory represents mathematical reason’s shift from a purely theoretical attitude to a phenomenological one. With this shift in focus the well-delineated border between philosophy and mathematics, that is to say the strict division of labour as envisaged by Husserl, needs to be revisited. It is possible that in discovering the life-world, and the “groping entrance into this unknown realm of subjective phenomena” which it made possible, Husserl in fact moved too rapidly onto what he considered lower-level questions.¹¹⁵ This is to say that there is a discovery-concealment at play in *FTL*, for example, when Husserl concludes that his expedition from traditional logic to transcendental logic has ultimately resulted in the need for all logics to be grounded by “a genuine mundane ontology”.¹¹⁶ But in discovering this perhaps ultimate phenomenological task, Husserl has in fact concealed the need for another important project and that is the critique of logical reason. As discussed in the previous chapter

¹¹³ Lawvere, “Tools for the Advancement of Objective Logic,” 43.

¹¹⁴ Husserl, *FTL*, 258.

¹¹⁵ Husserl, *Crisis*, 161.

¹¹⁶ Husserl, *FTL*, 291.

Husserl has of course outlined this task very clearly but the question is whether the work, which would actually succeed in carrying it out, has in fact been done. By taking a closer look at some of the aspects of category theory it will hopefully be possible to show that in fact it has not and that Husserl, guided by the formal ontology of his time, mistakenly assigned this critique to philosophy instead of mathematics.

§20 Categorification and de-formalisation

As John C. Baez and James Dolan put it, the concept of categorification, despite being fully clarified only recently, has been “lurking in the collective subconscious of mathematics for over a century”.¹¹⁷ It is the mathematical process by which set-theoretic concepts are generalised to those of category theory: elements become objects; functions become functors; isomorphisms give way to adjoint functors. Baez and Dolan offer the example of the category of finite sets which is simply a categorification of the set of natural numbers. For Baez and Dolan though, modern mathematics has been defined by its tendency for “pretending that categories are just sets”.¹¹⁸ That is to say that the entire history of mathematics could be summarised as a “de-categorification”, where morphisms have been completely forgotten and isomorphism has been treated as true equality. This Baez and Dolan clarify with a parable from ancient times.¹¹⁹ Here the shepherd, wanting to see if two herds of sheep are isomorphic, sets about matching sheep from each herd until there are no sheep left. One day however there is a shepherd who, in the search for increased exactness, decides to count the sheep in the first herd, i.e. show it to be isomorphic with a particular set of numbers, and then count the second herd of sheep, i.e. show it to be isomorphic with another particular set of numbers, before finally comparing those two sets of numbers to see if the two herds could be said to be equal. That is to say, this shepherd has invented counting and, as a result, de-categorified the category of finite sets to the set of natural numbers. They have in other words transformed two collections of objects into two sets of numbers. So it could be argued that, for Baez and Dolan, mathematics starts idealising long before Euclid. In an ordinary act of

¹¹⁷ John C. Baez and James Dolan, “Categorification,” in *Higher Category Theory: Proceedings of a Conference on Category Theory, March 1997, Northwestern University*, ed. Ezra Getzler and Mikhail Kapranov (Providence, Rhode Island: American Mathematical Society, 1997), 1.

¹¹⁸ *Ibid.*, 2.

¹¹⁹ *Ibid.*

counting the vagueness of isomorphism is replaced with the exactness of equality but at the same time there is a richness of information that is lost forever. Decategorification destroys structure, or perhaps more accurately, it destroys the act of structuring. Armed with the set of natural numbers, which now presents itself as pregiven, the working mathematician carries out their mathematical operations without ever paying heed to the fact that this set is indeed a categorial formation, it is the product of an intentional act. Now the phenomenologist can easily recognise the transformation in meaning that takes place in this idealisation: the mathematician, in their technical work, takes a categorial formation as ultimate substrate, takes an intentionally formed tool for measurement for the measured itself. As Husserl puts it, the theory of cardinal numbers has simply to do with the numbers themselves; the theory of ordinal numbers has to do with ordered sets and their form.¹²⁰ That is to say, as formal theories, they have to do with collections and not with acts of collecting. Decategorification, initially a “stroke of mathematical genius”, has become “dumb habit”.¹²¹ What Baez and Dolan are suggesting is that, completely blind to the scientific genius of their technology, the mathematician has been plodding away at their technical work with no regard for its historicity. This, with good reason, sounds very familiar, and it is worth also briefly noting that this whole way of conceptualising number, by showing its origins in the enumeration of a totality of objects, bares an uncanny resemblance to the work carried out by Husserl on the same topic in his very first book *Philosophy of Arithmetic* – the biggest difference being that where the mathematician says “finite set” Husserl chooses to say “collective combination”.¹²² Now while the mathematician has not been equipped with the methods to investigate the intentional implications of this categorial formation, and in fact has been completely blind to the possibility, the phenomenologist most definitely is well-prepared for such a study. But they are not alone; not anymore at least. With the dawn of contemporary mathematics, and the birth of the category theorist, phenomenology must now contend with a mathematics that can inquire into the intentional constitution of its own objects – conceptual mathematics has started to take form.

¹²⁰ Husserl, *FTL*, §56.

¹²¹ Baez and Dolan, “Categorification,” 2.

¹²² Husserl, *Philosophy of Arithmetic*, 21.

In *Ideas* Husserl very briefly defines the process of de-formalisation as the “filling out” of “an empty logico-mathematical form or a formal truth”.¹²³ Despite the fact that Husserl only ever specifically mentions de-formalisation this one time, Burt C. Hopkins actually argues that it is a fundamental concept underlying the whole of Husserl’s phenomenology.¹²⁴ What Hopkins is essentially suggesting is that, for Husserl, the phenomenological call to action to “go back to the ‘things themselves’” is nothing more than an incitement to return to a study of concrete phenomena by way of a de-formalisation of the formal-ontological “anything whatever”.¹²⁵ Read in this way the critique of logical reason could be described as an attempt to de-formalise formal ontology’s highest genus “category” and as a result ground the whole formal-logical enterprise, and the unity of its region which is devoid of material content, in the material *a priori* of the life-world. For Husserl the pre-predicative world of experience is one necessarily filled with individual objects and it is these indeterminate things that act as the ultimate substrates for all acts of cognition.¹²⁶ These individual objects would be then the presuppositions successfully clarified by way of the *epochē* of formal ontology – i.e. they are brought to light by a process of de-formalisation. Following on from this claim, and with consideration of the above discussions inspired by Baez and Dolan’s work, this could be extended to suggest that this process of de-formalisation is akin to what conceptual mathematics would call a “categorification”. There is however admittedly a problem with this argument that presents itself both immediately and conspicuously. In the above example of moving from the set of natural numbers to the category of finite sets it would seem as if, even though it could be argued that this is indeed an act of de-formalisation as envisaged by Husserl, this act of categorification does not manage to escape the fact that it is restricted to formal-ontological genera. That is to say that categorification does not seem to be able to get past the genus of category in order to reach the materiality of concrete experience. Even if it can be argued that category theory is no longer a formal mathematics in the sense outlined by Husserl can it truly be said that conceptual mathematics manages to breach the limits of the region accessible to an

¹²³ Husserl, *Ideas*, 26.

¹²⁴ Burt C. Hopkins, “Deformalization and Phenomenon in Husserl and Heidegger,” in *Heidegger, Translation, and the Task of Thinking*, ed. F. Schalow, Contributions To Phenomenology 65 (Springer Netherlands, 2011), 49–69.

¹²⁵ Husserl, *LI*, 167.

¹²⁶ Husserl, *Experience and Judgment*, 26.

analytic *a priori* science? So in some sense Hopkins's claim both supports and threatens the very aims of this paper: it helps elucidate the similarities between categorification and de-formalisation but at the same time puts into question whether or not starting from formal-ontological objects can indeed lead back to concrete experience.

§21 The “filling out” of formal-ontological objects

One way to defend the motivations of this paper, and the possibility of category theory being representative of a transcendental mathematics, would be to place the onus of any failure upon Husserl's own phenomenology. That is to say that it could be possible to say that while there is an undeniable congruence between categorification and de-formalisation that it is with Husserl's own attempts to “fill out” formal ontology's “anything whatever” that the blame lies for any inability to reach the pre-predicativity of immediate experience. Actually, this is exactly what Hopkins does when he claims that Husserl's attempt to ground formal ontology upon the individual objects of concrete experience fails on account of the conflation of different concepts of formality.¹²⁷ Now it should be mentioned that Hopkins does not ignore the fact that Husserl himself very clearly delineates generalisation and formalisation as being both species of universalisation which, at the same time as their genera are one and the same, are to be considered “totally different” from one another.¹²⁸ For Husserl, formalisation is the emptying of material content and the production of exact essences while generalisation allows for the discovery of morphological essences, and this is the difference, fundamental to phenomenology, between idealisation and ideation.¹²⁹ Hopkins is most definitely not unaware of this but his point is that, despite this delineation, Husserl erroneously insists that both forms of universalisation are of necessity grounded upon the intuition of individual objects. So while it may be true that the morphological essences with which phenomenology deals can be grounded upon the nourishing soil of the life-world, the same cannot be said of the ideal essences of the mathematical sciences. It could be argued that this relates back in some way to the view espoused by Carr that the concept of the life-world is at the very least ambiguous. Here Hopkins obviously

¹²⁷ Hopkins, “Deformalization and Phenomenon in Husserl and Heidegger,” 55.

¹²⁸ Husserl, *Ideas*, §13.

¹²⁹ *Ibid.*, 167.

understands the life-world as a purely perceptual world completely lacking categorial formations. But the ambiguities pointed out don't succeed in refuting Hopkins's opinion but perhaps simply highlight the need for further investigation into what kind of world the critique of ontology can help to discover. And despite the fact that Hopkins's view of Husserl's project potentially closes off the ontological way into the thematisation of transcendental subjectivity it is impossible to completely disagree with his claims. That is to say that on top of this problem with the conception of the life-world, this paper's thematisation of conceptual mathematics also helps bring to light a potential oversimplification in Husserl's conception of universalisation.

As Goldblatt describes it, category theory allows for the study of the invariant notions of mathematics by "an abstract formulation of the idea of mathematical isomorphism".¹³⁰ This could be seen as just alternative way of saying that conceptual mathematics involves the search for common features of structured mathematical forms, but that this commonality is not itself to be regarded as a structure. This is of course also in line with the idea that was outlined at the beginning of this chapter that the mathematical objects with which category theory deals, arrows, are not of the same species as the mathematical objects upon which they act. Goldblatt provides his readers with a handy table of some examples of categories which has been reproduced, in part, below as figure 2.

CATEGORY	OBJECTS	ARROWS
Set	all sets	all functions between sets
Finset	all finite sets	all functions between finite sets
Nonset	all nonempty sets	all functions between nonempty sets
Top	all topological spaces	all continuous functions between topological spaces
Vect	vector spaces	linear transformations

Figure 2. Select examples of categories, their objects and their arrows¹³¹

From this the fact that was discussed at the beginning of the chapter, that all categories involve objects and arrows, becomes a little clearer but what is most

¹³⁰ Goldblatt, *Topoi*, 98:42.

¹³¹ *Ibid.*, 98:23.

important to understand, as Goldblatt points out, is that the commonality that all categories share – the very thing that makes them categories – is the way in which their arrows behave.¹³² So rather than focusing on the internal structure of its objects category theory thematises the way in which external relations are constituted. By doing so the conceptual mathematics that has flourished since the second half of the 20th century can trace the transformation of mathematical objects in a way that the formal mathematics of Husserl’s time couldn’t even dream of. This contemporary breed of mathematics can account for the act of doing mathematics, it is able to focus attention on the way in which the mathematics community works with their mathematical objects. This means that it is able to de-formalise its thematic objects and trace their genealogy much in the way that Husserl himself was trying to do. This was exemplified above with the example of the categorification of the set of natural numbers to the category of finite sets. But one important detail was occluded and that was that it is, for example, also possible to de-formalise the natural numbers into the category of vector spaces. So generally there is a choice between different objects involved in this process and this relates back to Hopkins’s criticism in a way. Where Husserl saw this de-formalisation as necessarily being able to trace the constitution of formal objects back to concrete experience, this example of categorification helps to highlight that it perhaps just leads to new forms. Hopkins summarises this by saying that Husserl’s method gets stuck in the “mode of the ‘objectification’”.¹³³ This is potentially because Husserl, despite clearly delineating generalisation from formalisation, had not been able to fully clarify its relationship to de-formalisation. This is very much related to the fact that the power of adjointness, which makes the genealogical study enacted by category theory possible, is the fact that it is one of approximation. It is interested not with a relationship of equality, or even equivalence, but one of similarity. Marquis describes the concept of adjointness as an attempt to “get as close as possible” to the mathematical objects that make up the possible origins of a given structure.¹³⁴

If any success has been achieved thus far in showing the congruence between de-formalisation and categorification then it should become evident that de-formalisation appears to be, more correctly, a version of generalisation. The formal-logical

¹³² Ibid.

¹³³ Hopkins, “Deformalization and Phenomenon in Husserl and Heidegger,” 51.

¹³⁴ Marquis, *Geometrical Point of View*, 152.

formations that are made thematic by de-formalisation are already emptied of all material content and the “filling out” that Husserl refers to is in fact just another way of saying that it is possible to find other forms that that the object in question could have originated from. So these origins, or categories as the conceptual mathematician calls them, have their thematic objects as commonality. The already formalised object is traced back to more generalised forms, and as it is already empty this process can obviously not be one of increased formalisation – there’s no content left to empty – so must be considered instead an act of generalisation. This is in part why category theory is jokingly referred to as “general abstract nonsense”.¹³⁵ Conceptual mathematics then, even though it deals with categorial formations, is not a formal mathematics (or at least not a formalising one). This argument is somewhat supported by Null and Simons – at the same time as they go part of the way in helping to refute it. This is because while they also find a level of confusion in Husserl’s investigation of conceptualisation, they place the blame on a conflation between generalisation and specification – concluding that rather than being a process of generalising, ideation is an act of materialisation.¹³⁶ Regardless, there is undoubtedly some obscurity surrounding the intentional analysis of conceptualisation that calls for further investigation, and the difficulties that Husserl obviously had in clarifying this concept seem to stem from the inability of the formal ontology of his time to account for its own constructions. As already elucidated at length above, Husserl saw the task of critiquing logic as falling within the region of phenomenology but it is hopefully by now possible to argue that it was at the same time not equipped for this very task. This means that Husserl’s delineation of mathematics and philosophy was in some sense faulty and to clarify this point it would prove helpful, somewhat ironically, to further elucidate his almost prophetic vision concerning the theory of manifolds.

§22 The theory of manifolds revisited

By generalising the various mathematical discourses into objects and arrows category theory can, rather than merely focus on internal structures, better thematise the external relationships that account for the formation of mathematical objects. What this level of generalization means is that it becomes possible to carry out interpretations between otherwise disparate theories and discourses. It may help to

¹³⁵ Marquis, “What Is Category Theory?,” 221.

¹³⁶ Null and Simons, “Manifolds, Concepts and Moment Abstracta,” 478.

conceptualise somewhat just what category theory is by adding that if abstract algebra could be called the study of the languages of algebraic structures then category theory could be referred to, metaphorically of course, as a translation between them. As Krömer points out it is one of the “slogans” of category theory that different domains can have unexpected connections, and this goes back to the very origins of category theory in the fact that Eilenberg and Mac Lane began to collaborate due to the surprising similarities between their respective fields of algebraic topology and algebraic number theory.¹³⁷ There is an obvious affinity then between category theory and Husserl’s theory of the manifolds, and it is precisely this relation which led to the rebuttal of da Silva’s claim earlier that Husserl was himself opposed to mathematical liberality. Husserl’s conception of the theory of manifolds is of its essence concerned with the discovery of these kinds of connections between otherwise seemingly remote theories and if, as Husserl claims, the formal mathematics of his time was a partial realisation of this “idea of a science of possible deductive systems” then it is argued here that category theory brings this idea even closer to its fulfilment.¹³⁸ It is worthwhile pointing out that Husserl actually developed the concept of the definite manifold in an attempt to come to terms with how “imaginary” numbers can be used successfully in operations concerning a formally defined manifold, the natural numbers, to which they do not logically belong.¹³⁹ This helps to illustrate the point that the theory of manifolds, as conceived by Husserl, only managed to elucidate the possibility of comparing formal structures belonging to the same region of study, in this case number theory. What is being suggested is that while he had the genius to reveal the idea of the theory of manifolds, Husserl at the same time concealed from himself just how this concept could be fulfilled, and this discovery-concealment concerns his delineation of phenomenology from mathematics. So it is necessary to agree with da Silva on another matter, and that is the way in which he describes Husserl’s delineation of formal and applied mathematics as “mathematically irrelevant and philosophically misleading”.¹⁴⁰ It would seem that Husserl believed that simply by following its formalising actions to their logical end point, without any concern for its practical purpose, mathematics could achieve its goal of the

¹³⁷ Krömer, *Tool and Object*, 51f.

¹³⁸ Husserl, *FTL*, 92.

¹³⁹ *Ibid.*, 97.

¹⁴⁰ da Silva, “Mathematics and the Crisis of Science,” 357.

development of all true theories. But it must be noted that, as Lawvere points out, the “pragmatist theory of teaching only skills” should be seen as an aborted project that never reached the goals it had set out before it.¹⁴¹ Without any philosophical underpinnings the student of mathematics can never decide which procedure is right for their desired application. In order to remedy this Lawvere suggests that what is needed is a “sober appreciation of historical origin of all notions”.¹⁴² That is to say that the fact that category theory makes thematic the subjectivity of formal mathematics, and even hopes to investigate the historical sedimentation of its concepts in a way Husserl thought only possible for his own phenomenology, is what grants it the possibility to fully realise this system of deductive sciences. Relating this back, even further, to the contention that phenomenology should perhaps dare to intrude upon the methodology of mathematics, this would be offered as evidence to that effect. So it could be argued that Husserl was in fact obscured by the division of labour that he, inspired by the limitations of formal analysis, advocated and that in some sense by taking a transcendental turn mathematics – in the form of category theory – was itself able to extend its region into that of phenomenology and to subsequently realise the theory of manifolds as described by Husserl.

¹⁴¹ Lawvere, “The Category of Categories as a Foundation for Mathematics,” 12.

¹⁴² Lawvere, “Taking Categories Seriously,” 177.

EPILEGOMENA

The final chapter will now summarise the paper in a set of concluding remarks. This will hopefully help to finalise the opinion espoused in this thesis that the development of category theory helps shed new light on the somewhat nebulous relationship between phenomenology and mathematics.

§23 Conclusion

If this paper has failed in its attempts to awaken the possibility of conceptual mathematics, and category theory, as transcendental science it is at the very least hoped that it has still succeeded in indicating the need for the clarification of a naïvely inherited prejudice. That is to say that there is a distinct need for the elucidation of the full sense of the division of labour between philosophy and mathematics – which at all times remains operative in Husserl’s phenomenology. If the thought of mathematics encroaching upon phenomenological work, complete with its thematisation of transcendental subjectivity, causes an almost visceral recoiling in the reader then it must be clarified just what this revulsion means. Ironically enough, what is being suggested is that where there is a purely instinctive resistance to the possibility of mathematics entering into the phenomenological domain then there is the distinct possibility that a full suspension of belief in the objective sciences has not been enacted. Any unclarified resistance to formal ontology ratifies its very power over phenomenology. The risk is that by instinctively turning away from mathematics there is in actual fact a “confounding of the ego with the reality of the I as a human psyche” at play.¹⁴³ This confusion (perhaps the ultimate transcendental sin) arises because there has not been attained the genuineness promised by the transcendental reduction and its associated *epochē* of categorial forms. To avoid these pitfalls the delineation between philosophy and mathematics with which phenomenology operates must at the very least be made thematic.

Hopefully this paper has managed to do more than just indicate the need for a re-delineation of mathematics and phenomenology and has indeed elucidated the possibility of a transcendental mathematics which could – together with a transcendental psychology – investigate problems relating to intentional constitution

¹⁴³ Husserl, *FTL*, 230.

and perhaps even help towards clearing some of the obscurities concerning Husserl's problematic conception of the life-world. It is hoped that it has been shown that the material *a priori* science Husserl envisaged was deeply rooted in the limitations of formal-mathematical work that was going on around him during his time at Göttingen. By widening his historico-critical investigation to include the development of conceptual mathematics the aim has been to show that the concept of the definite manifold, which much of Husserl's phenomenology is built around, can no longer be seen as being coherent with the sense of mathematics and that as a result the critique of logical reason, which Husserl saw as belonging to philosophy, may be better assigned specifically to category theory. The most obvious rebuttal to these claims would of course be to accuse this paper of objectifying transcendental subjectivity in exactly the same way that any naturalised phenomenology does – and problems relating specifically to this issue have been, while not solved, at least raised during the course of the foregoing discussions. No defence will be offered here because until the question of the delineation between mathematics and phenomenology is resolved this paper is in some way barred from doing so. Any attempt to bring these claims to intuition would unavoidably involve doing mathematics, but if the formal/material divide was not first re-imagined then these attempts would be anyway simply cast aside as philosophically meaningless symbolic work.

What is meant by all this is that the pre-existing division of labour must not be simply clarified but rather thoroughly dissolved and thought anew. Philosophy perhaps poses questions that it does not possess the ability to answer; mathematics may be answering questions it does not have the insight to understand. But in order to investigate these problems pre-given prejudices would need to be questioned and the possibility of inter-communal communication would need to be addressed. The borders between mathematics and phenomenology would need to be re-mapped – if not entirely discarded. In closing, it would be more than fitting to borrow the words of Lawvere in order to say that the contemporary situation in which phenomenological science finds itself “will require that philosophers learn mathematics and that mathematicians learn philosophy”.¹⁴⁴

¹⁴⁴ F. William Lawvere, “Categories of Space and of Quantity,” in *The Space of Mathematics: Philosophical, Epistemological, and Historical Explorations*, ed. Javier Echeverria, Ibarra Andoni, and Thomas Mormann (Berlin: De Gruyter, 1992), 16.

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