Exploring theoretical models with an agent-based approach in two sided markets

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Abstract

With increasing computational power and more elaborate software comes greater opportunities to complement traditional research methods with alternative methods. In this paper we argue for why the area of two-sided markets could benefit from this alternative approach and attempt to implement a theoretical model in an agent-based framework. By first replicating the theoretical findings in this framework we expand the model in increments in different directions through introducing different set of heterogeneity and behavioral limitations on our actors to see how the theoretical model develops. Only changing the model in increments found the analytical outcome to be robust for many of our changes, in this regard we have not managed to successfully take advantage of the full potential of the agent-based framework.
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1. Introduction

The growth of two-sided markets, technology and competition has reshaped the landscape for a lot of companies in today’s economy. Not only has new technology created entirely new products and services such as music- and media streaming services, ride sharing applications, online travel agents amongst numerous other business creations. It has also affected older industries with two-sided characteristics such as magazines, payment systems and newspapers to adapt and to a large extent abandon old practices\(^1\) of conducting business. Although the business practices of two-sided markets can hardly be counted as something new, with the potential of larger coverage, increasing internet speed and better technology many more business ideas which carries with them a coordinating function between two different types of users have become viable practices.

The traditional notion of a firm is one where a number of inputs are produced into an output to go for sale. Two-sided platforms on the other hand are peculiar in the way that a successful platform must balance the needs and wants of two sides. It has a coordinating function making it easier for the two to interact and transact with each other. As a member of a one side of the platform I care about the number of providers on the other side. The prevalence of so called network effects\(^2\) has not surprisingly received a lot of interest by scholars in economics in the field of industrial organization, with a large literature now available on the subject.

The number of studies has exploded in recent years, most of them being theoretical models with different levels of complexity. They focus on how competition and equilibrium are expected to unfold given certain assumptions and how an outcome might change with changes in the assumptions.\(^3\) The empirical papers have however been lacking behind in development (Sririam et.al 2015). The authors maintain that a potential cause for this might be the higher requirements of data necessary to conduct robust empirical studies (Ibid: 3). Reviewing and outlining several research opportunities in the field the authors highlight potential areas for empirical research to: (1) test the theoretical predictions of analytical papers in the area (2) collaboration between researchers to make available and use of new platform data and (3) methodological challenges of analyzing platform data (Ibid: 2-3).

This leads us to the purpose of this paper. My contention is to approach the study of two-sided markets using an alternative framework and method of conducting economic research. I will try to argue that the peculiarities of two-sided markets could benefit from a complementary approach beside analytical theorizing and the common tool of regression analysis and hopefully be able to provide an example how this alternative approach could be used to shed further insights into the theoretical predictions done by analytical papers in the area, which often force

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\(^1\) I am here thinking of offline distributing of newspapers/magazines, paying with cash, checks and so on.

\(^2\) Also known as externalities, feedback loops, vicious- or virtues cycles, cross-network effects.

\(^3\) I.e. allowing for multihoming behavior, introducing competition for general explorations of two sided markets and industry specific assumptions, whenever a more narrow approach is examined. Meyer (2012: 57-58) make the same observation.
constraining assumptions to invoke closed-form outcomes in an area empirically proven to be very dynamic in its nature. In a way one could argue that we take upon the first suggestion of Sririam et. al (2015) but in another framework, that of computer simulations. This alternative approach could also circumvent the difficulties in the higher data requirements necessary to conduct standard quantitative techniques.

Using the software NetLogo I implement a relatively simple theoretical model into an agent-based framework. By managing to replicate the closed-form solutions analytically derived by the authors Gabszewicz & Wauthy (2005) the intention is to expand the model by following some of their suggestions in their final remarks regarding limitations in their model and suggestions of how to expand their work. The intention is to depart from notions of full information and perfect rationality to see how outcomes might change with bounded rationality.

The potential of agent-based models lies in setting simple rules, different incentives and establish heterogeneous interactions between agents, firms, consumers, producers etc. We could exogenously or endogenously model perfect- to imperfect behavior in their utility- and profit maximizing choices. This approach to studying concepts and important characteristics related to two-sided markets are fairly new and manages – in the authors view - a more satisfactory way validate and dynamically represent the multiple static outcomes derived by the theoretical papers in the area as well as with less heavy data requirements depict the empirical outcomes observed in, amongst others, the video game console industry.

The rest of the paper is structured as follows. In section two I will provide a general description of what two-sided markets are and their key characteristics much focused on in the academic literature followed by a selection of studies on papers dealing with two-sided markets in a simulation and agent-based framework and lastly a paper on dynamic pricing. Section three introduces the theoretical model our simulations will be based upon. Section four and five will contain the heart of the essay – here a short description of the program is presented, in what ways we intend to tweak the underpinning theoretical model as well as an explanation of the models to be run in detail and the results of our simulations. Section six will mainly be a discussion of limitations, validation- verification and replication processes when conducting simulations and lastly a summary will follow.

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4 The relative ease of which firms can establish themselves in new digital products, collaborations between complementary software, importance of networks and so forth.
6 Assuming that the authors bias has not clouded the search process of previous empirical studies, a large share of the lacking empirical research on two-sided markets seem to be largely focused on the video game industry.
7 One could think of the section as a discussion of the robustness of the results for the cases considered.
2. A general description, summary of important characteristics and previous literature of two-sided markets.

Two-sided markets have been defined in various ways by the major contributors of the area in the early 00’s. As there seem to be a great deal of similarities in the different definitions of what constitutes a two-sided market and what markets in the economy that actually fall under this umbrella my contention here is not to delve too far into the details of its definition. Only a general description will be provided with the inherent important characteristics which might be new to readers who are not familiar with the literature.

A platform creates value through maintaining an intermediate position bringing two different users groups, who might benefit from each other’s presence, together for potential trade. Common examples of two-sided markets in the literature are video game consoles, payment systems, shopping malls and newspapers. Newer examples include music- and video on demand services, ride sharing applications and online comparison websites. Each of these businesses maintain an eco-system consisting of two different user groups which must be kept satisfied for further survival, granted a certain level of competition exists. Companies providing gaming consoles must for example maintain gamers on one side and software developers on the other, music streaming services requires consumers on the one side and music labels/artists on the other.

Equilibrium outcomes in models of two-sided markets often have special characteristics. We could summarize what distinguishes two-sided markets from a regular input-output firm with five key characteristics (Meyer, 2012: 40-42). The four first are what the previous literature in the area have focused much of their attention on (1) Finding the optimal balance of prices between the two sides. As Rochet and Tirole (2006) made clear, price structure matters. There are few firms in equilibrium, depending on the strength of network effects (3) Sensitivity of initial conditions and the role of path-dependency. (4) Multi-homing behavior (5) lock-in effects. The common denominator in most of these characteristics are the presence of network effects, the importance of coordinating and decreasing potential transaction costs for the two or more groups on opposite sides is crucial for two-sided markets.

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8 The interested reader can consult: Rochet & Tirole (2006), Evans (2009: Chapter XII), Rysman (2009). A recent review dedicated to the definition of two-sided markets can be found Ardolino et. al (2016)
9 See for example the literature review by Ardolino et. al. (2016: Table 1, pp. 3-4)
10 Platform can be thought of as a fair, shopping center, streaming applications, or comparison websites.
11 In defining a two-sided market Rochet & Tirole (2006: 658) clarify that one requirement of two-sided markets are that the price structure, that is, the profit maximizing price on the two sides of the platform is asymmetric.
12 Due to space and lack of significance for the simulations to come in this current paper the author settles with a short explanation of multi-homing behavior here: It is simply the behavior of a consumer adopting two or more platforms providing the same service/product. Think of, i.e. someone using both the Uber and Lyft application.
Network effects

The most important property distinguishing two-sided markets are the concept of network effects. Network effects are usually distinguished in two categories, direct- and indirect effects (Katz & Shapiro, 1985: 424-426; Oz, 2011: 120). An example of the former is when the value of a platform increases with the absolute number of users, a typical example of platforms that derive their value from direct network effects would be social media. Having, for example, a larger share of friends and family using the platform enhances the value of said platform as the accrued benefits get larger, or in economic terms one’s utility increases. The telephone is often used to exemplify direct network effects - the value of the technology increases as a function of the absolute number of users as you can reach more people with it. The second category of network effect is when consumers are concerned with either complementary users or software. Using a dating app one would be concerned with how many of the opposite gender that uses the platform and to a lesser extent how many of the same gender that have adopted the platform, in this case the direct network effect might even affect the platform negatively to some small extent. Streaming sites such as Netflix and Spotify one would be concerned with the supply of consumable media available.

Accepting that at its core two-sided markets most valuable assets are the network of users from each side dedicated to the platform we could derive its effect on (1) – (5). A successful platform must adhere to keeping both sides satisfied as the potential decline of one side could strongly effect the number of participants on the other side. Triggering a loss of consumers on one side would mean that the negative dependent effects would fuel each other in a vicious cycle until a potential break down in the network.14

Price structure

How prices are set by multisided platforms has drawn a lot of attention because of the influence network effects have on the potential success of the platform itself. Industries clouded by strong network effects where one side of the market is more sensitive in their demand than the other often leads to asymmetric pricing by the consolidating platform. This is also one of the criterion Rochet and Tirole (2006) lay forth for an industry to be considered a two-sided market. In their earlier paper (Rochet and Tirole, 2003: 1009) refer as far back to Ramsey (1927) in order to explain the nature of optimal pricing behavior in two sided markets.15 To maintain a successful platform prices should be set asymmetrically in favor of the side that is more sensitive to demand. For example, we do not pay anything for comparing products on online websites such as flights, accommodations or durable goods. In these cases as each purchase is traceable a fee is instead taken by the comparison website on the other side of the market i.e. in this case the flight

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13 Indirect network effects are also referred to as cross-network effects, that is, the number of agents on the other side are of significant importance. Direct network effects refers to agents on the same side as oneself, as this will be shown to have an impact as well on the attractiveness on a platform.

14 Network in this case is usually referred to the two or more participating sides of the platforms business.

15 The insight I am referring to here is the notion of setting the markup (or even subsidy) to the side most sensitive in their demand. I believe this to be the application (b) in (Ramsey, 1927: 59)
company / accommodation provider / store in which the purchase was made. The increasing feasibility of tracing each transaction made on a platform leads us to the second peculiar characteristic on pricing. A platform has to optimally select four different set of prices; each of the two sides can have both membership- and transaction fees. How one should balance these four in the presence of network effects could be argued to make the profit maximizing decision relatively difficult for companies.

One last observation on pricing is the dynamic price-setting behavior present under the phase of growing the business. The early literature on two-sided markets only aiming at explaining interesting business conducts of these practices took note that companies often go so far as to subsidize the two sides in order to harness a network and given its success only after some time starts to raise prices (Evans, 2003: 200-205). This has in recent years been an issue for anti-trust and regulation as the relevancy of traditional concepts such as predatory-pricing and pricing below cost have been called into question given the necessity of creating a network of sufficient size in order for the customers on the opposite side of the platform to find value in using their product/service (Evans & Schmalensee, 2005: 29).

**Sensitivity of initial conditions, path-dependency and few firms in equilibrium**

The other strand of literature Tirole and Rotchet (2006) uses with Ramsey pricing to characterize two-sided markets are on network externalities. Two iconic papers come to mind, one is the persistence of the QWERTY-keyboard by David (1985) and the other is the study by Arthur (1989) on competing technologies.16

The short paper by David (1985) shows the sensitivity of market equilibrium to initial conditions when it comes to products / services exhibiting network effects. Despite the fact that the QWERTY-keyboard was developed to slow down typing in order to avoid keys jamming17 it has none the less managed to maintain its dominance to the present despite the very reason for its development is no longer an issue. David outlays that even though no technological or institutional advantages was given the QWERTY-keyboard and competition of alternative arguably more effective designs were developed (Ibid : 334), none of the later designs managed to dethrone the QWERTY-keyboard as the standard to be used. The reason given for this, to my understanding, is that during the short time period when adoption of the keyboard occurred on a larger scale, this was done on multiple levels in society and involved not only organizations procuring technology with the QWERTY-keyboard, but the fact that people have to be trained in using a keyboard with a given design. Organizations, both public and private need/want operators with experience, and thus this interrelatedness creates a complex system (Ibid: 334) which no single firm has an incentive to opt out of. Given that the QWERTY-keyboard is less

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16 These two studies are chosen on the basis of the theoretical background presented by Meyer (2012:5)
17 As explained by David (1985: 333) when using the typewriter, if one were to type too fast there was a strong likelihood of the keys getting jammed
efficient than other potential designs\textsuperscript{18}, the private cost for a company would outweigh the potential benefit of having to train employees in a different system, even though the world – the social benefit - as a whole could be argued to be more productive with another standard.\textsuperscript{19}

Having developed railway gauges of certain standard measurements could also be seen as a kind of initial conditions future railways has to pay regard to, given the railway network established is of sufficient size. If some alternative gauge measurement were to be proven more advantageous but unable to be compatible with the already established network one could argue that initial conditions limits potential efficiency gains by having to adhere to the old standard. Puffert (2002) goes to show the importance of initial conditions, contingent events and path-dependence for the development in the standardization of railway track gauges (Ibid: 311) and how these conditions and events rather than optimization are the result of what we see today.

The study by Arthur (1989) sheds light on path-dependency when increasing returns to adoption is present.\textsuperscript{20} In a very simple model two technologies are assumed to compete against each other and where potential consumers derive value from the fact that other users have adopted the same technology as well. Potential users adopt the technology in sequence, and each potential user is assumed to have a preference of one technology over the other. Assuming no one has yet committed to any of the technologies one would just pick the technology most preferred. Arthur (Ibid) goes to show that if the network effects are sufficiently strong, despite the fact that some consumers prefer one technology over the other, if one of the two technologies – by random chance\textsuperscript{21} – manages to grow in sufficient size then the network-effect would dominate the “preference-effect” and despite the fact that some consumers who otherwise would prefer the technology with less users now would opt to their less preferred choice because of the large amount of users already committed to that technology.\textsuperscript{22} At this point the technology could be said to have been “locked-in” as well.

Sensitivity of initial conditions and path-dependency can thus easily give rise to the empirically observed fact\textsuperscript{23} that often the number of firms active in any given two-sided industry are few in numbers. From the perspective of the agents adopting the technology, or platform in our case, the incentives remains quite clear. It is easier, more convenient and less costly to search for a certain product on one platform than to have to visit several. Having one or two platforms providing the

\textsuperscript{18} David (1985) is only the canonical paper, many papers and discussions have followed – some which question the inefficiency of the QWERTY-keyboard. An elaborate exposition can be found in Meyer (2012: 11-26)

\textsuperscript{19} In some sense the incentives are not present at the micro level, although the benefits are clear at the macro level.

\textsuperscript{20} It is in these cases that Arthur (1989) models that tipping might occur and one technology corners the market. Under constant or diminishing returns of adoption two or more technologies are shown to be able to share the market.

\textsuperscript{21} Arthur (1989: 118) uses “historical events” or “chance” to explain the fact that there lies uncertainty in the adoption process beyond the knowledge of any observer.

\textsuperscript{22} It is important to point out here that the model he uses is stochastically dynamic; of multiple possibly equilibria under increasing returns of adoption, one may predict that technology A or B will corner the market but not which one, we have non-ergodicity (Ibid: 122) and this is in turn decided by the path-dependency of how users are adopting the technologies present.

\textsuperscript{23} I use the term in the sense of “stylized facts” or “casual empiricism”
service of ride-sharing services means less time to wait for a vehicle – the market could be said to thicken. Similar incentives could be argued to be present on the other side of the platform as well, producers providing their products and services avoid the cost of potentially having to adopt several platforms, where their potential customers may be scattered. Of equal importance would be the economics of scale present, as with most markets in the digital sphere the marginal cost, or variable cost, is often low compared to the fixed cost of setting up the actual platform.

Finding the optimal asymmetric prices getting the two sides on board, managing and maintaining expectations of adopters and potential adopters, sensitivity of initial conditions and the inherent path-dependent process that is present in two-sided markets are the reasons why I follow Meyer (2012: 62-63) in agreeing that an analysis based on simulation is justified in the study of two-sided markets. Under these circumstances static theoretical analysis in this area seem to be less robust than the traditional input-output firms competing in a Cournot or Bertrand fashion.

Agent-based simulation studies in two-sided markets.

In arguing for a more dynamic approach toward the complexity and non-trivial dynamics of two-sided markets Heinrich & Gräbner (2015) embark their paper by scrutinizing the conditions necessary for an equilibrium outcome in the canonical model by Rochet and Tirole (2006) (Heinrich & Gräbner, 2015: 5). In this critique we find the lack of heterogeneous interaction between the two sides (Ibid: 5), fixed assumptions of network sizes by agents (Ibid: 6), combining membership- and transaction prices to one measure (Ibid: 6), which thus would imply no difference for the platform provider when considering to raise the prices. Increasing the membership fee or transaction price will affect the total network and agents differently depending on the usage/benefit of said platform and finally they contest the required assumptions of a unique equilibrium (Ibid: 7).

Heinrich & Gräbner (2015) consider three different decision making algorithms from the platform providers in their simulation model. Trying to impart the limitations of theoretical derivations and the benefits of agent based models (ABM) they use the Rochet and Tirole (2006) model as a benchmark to be simulated and compares latter models comprised of bounded rationality with the benchmark case with perfect information.

The cases and so-called “decision making” behavior from the point of the provider were (1) an optimization algorithm (2) Reinforcement learning (3) Reinforcement learning with satisficing. The crucial difference between (1) and (2) lies in alleviating the assumption of the provider’s knowledge of the distribution of benefits that the agents incur when and if adopting the platform, or in other words the notion of perfect information. As (2) still aims for the platform provider to maximize profits through multiple periods by experimenting with prices continuously the 3rd

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24 This is called “A competitive bottleneck” Rochet and Tirole (2006: 659). When consumers only adopt one of several platforms present this would force service/product providers to be present on all of them to reach every consumer, this alleviates competition on the producer side. One could think of it as the platform having monopoly on a subset of consumers single-homing their platform which producers want to reach.

25 For an overview of the cases considered see Heinreich & Gräbner (2015), pp. 14.
model caps the experimenting behavior by the platform provider through a “satisfaction” constraint.\textsuperscript{26}

The results provided from the simulations confirms the ability to replicate the theoretical model of Tirole and Rotchet (2006). By the first model the authors highlight the relatively lower entry fee that the smaller group has to pay (sellers) compared to the larger group (buyers). This is believed to be intuitive in light of the importance of building up the network, a critical mass of users is necessary (Heinreich & Gräbner, 2015: 17). Further when they try to simulate optimization with both membership fees and transaction fees the platform is often unsuccessful, leading to empty networks (Ibid: 17).

When imposing bounded rationality\textsuperscript{27} the authors observe that the simulation does not settle at an equilibrium but follows certain patterns, being more volatile and path-dependent compared to the optimizing price-setting behavior of case (1) (Ibid: 17). Further the results indicate that the bounded pricing behavior initiates selection by the agents in the simulations, with them choosing the platform with most favorable pricing conditions enhancing the attractiveness and revenue of the given platform (Ibid: 19).

In the comparison of the decision-making algorithms (2) and (3) the authors find that establishing and maintaining networks are more successful when the platform providers behave under some satisficing condition (algorithm 3), along with competition. This I believe is as they mention that with only one platform, in either pricing-decision of (2) or (3), the platform sets malicious prices. Likewise when no satisficing condition is present, it can lead to overshooting or undershooting in the set of prices, and with increasing cross-network effects this would obviously abrupt the number of customers adopting a certain platform, causing a breakdown in the network (Ibid: 19).

What is intriguing is to combine (Heinreich & Gräbner, 2015) simulation findings of under- and overshooting the set of prices when they only consider one monopoly provider or when no satisficing condition is present with Zhou (2012). Zhou (2012) concludes through using real-world data in the fifth-generation video game market and a counterfactual dynamic equilibrium model\textsuperscript{28} that overshooting in prices in one side of the market for Sega led to its demise in the video gaming console market (Ibid: 1, 4).

\textsuperscript{26} To my understanding this would mean that the pricing behavior of the provider is programmed to be aimed at i.e. covering costs, or a threshold of some profit margin to be satisfied.

\textsuperscript{27} These would be the cases where the decision-making algorithm implemented is (2) and (3). See figure 9 (Heinreich & Gräbner, 2015: 24)

\textsuperscript{28} To my understanding this is done through manipulating the prices and relevant variables in order to assess how competition and the profits for Sega would have developed, if they had managed a more “network-friendly” approach to their pricing.
With regard to the pricing decision Heinreich & Gräbner (2015) find that the platform providers charge more in membership fees toward the larger group (Ibid: 22) in all three cases, and the opposite in the case of transaction fees (Ibid: 22) when comparing the rational provider (1) with the bounded provider of (2) and (3).²⁹

The latter type of divergence of transaction fees which potentially could be overlooked in theoretical papers have some empirical confirmation to it, in favor of the bounded providers. Online travel agents and price-comparison websites in general, where the transactions can be traced, often charge the sellers whenever a purchase/booking is made, and they are arguably fewer in comparison to the number of consumers using these sites. Another not so intuitive example would be Spotify, each “transaction” could be each song played by the consumer, which cost them nothing, while the artist/label are compensated, a small amount, for each time their song is played.³¹

If one could summarize the previous study of Heinreich & Gräbner (2015) to shed light on the sensitivity of maintaining a network, difficulty of optimizing prices and finding a stable equilibrium by platform providers a second paper using an agent based approach by Huotari et.al (2016) provides complementary evidence on the importance of behavioral limitations with a focus on the agents, rather than the platform. In essence their paper tries to contradict the “winner takes all” argument that is prevalent in the literature of two-sided markets by assuming consumers to be selective attentive and locally biased in their adoption decisions.

Mainly they argue and make their case through simulations using different assumptions of agent behavior and combining real world data from the video game console industry. With the convenience of having the empirical market outcome as their validating counterpart the authors construct four different models to simulate in order to find which type of consumer behavior replicates the real world outcome with least errors.

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²⁹ The rational provider had a larger transaction price toward the larger group (buyers) while the opposite were the case for the bounded decision-making algorithms (2) and (3).

³⁰ This would be their previous critique of Rochet and Tirole (2006) bundling membership- and transaction fees to one price.

³¹ Consumers play the songs for free and the artist gets compensated for each play, making consumers relatively worse of in transaction fees. This is in line with the results that the larger side pays the higher price. I am sure however that the compensation structure is probably more elaborate than this.

³² Arthur B (1989) model with increasing returns to adoption can be considered a “winner take all” model. The study can also be seen as a “stab” toward the simplistic utility functions, where the network size matters (number of people) rather than the quality (the actual number of people one would care about using the platform)

³³ In short, selective attention is given the meaning that potential adopters of a certain gaming console perceive newer games to be of higher quality than older games, thus biasing the decision with respect to the utility of the different consoles toward the one which has the “freshest” games (Ibid: 7).

³⁴ The assumption of local bias is implemented through the so called small-world network theory. The important characteristic of this assumption is that potential adopters are highly clustered, with so called short pathways to each other (Ibid: 7). One could think of this assumption as biasing potential adopters of a certain video game console to depend more on friends, family and acquaintances rather than the absolute number of consumers for each console.
The assumptions made in the four different models are that the agents are either unselective- or selective attentive and either globally- or locally biased. Two of the four models thus contain the usual assumptions made in theoretical papers of two sided markets with agents behaving with perfect information. In their paper this would be *unselective attention*\(^{35}\) and *global bias*.\(^{36}\) The latter two models make the opposite assumptions, our attention is *selective* and that we are *locally biased*.

The two models that came to explain the empirical outcome with greatest accuracy, granted some optimal value parameterized for local bias and selective attention, was in fact the models implementing some type of cognitive restraint and biases for consumers (Ibid: 27). The authors conclude by invoking these characteristics as important factors in order to explain Playstation 3’s success as a late-entrant and eventual overtaking of the absolute number of adopters from the incumbent Xbox (Ibid: 32 – 33), despite it being on the market with one year’s head start (Ibid:22).

A last study relevant to examine in relative length is Nedelescu (2013). Using an experimental method Nedelescu explores the potential pricing behavior of a monopoly platform in a two-sided market environment. The main objective of the paper is to accrue knowledge for policy makers in anti-trust and regulation that strong network effects where one side might be highly more sensitive to price than the other facilitates asymmetric pricing and that this type of pricing behavior does not necessarily imply predatory-pricing or exploiting potential market power (Ibid: 5-6).

With no authority dictating how the monopoly is allowed to set the prices, both the extensive literature overview (Ibid: 2-24) and the experimental results indicate that it may very well be both profit maximizing as well as welfare enhancing to aim for the optimal price policy to be below cost for one side in some instances while extracting a higher price of the other. A comparison of these outcomes is then made in the paper as Nedelscu’s (2013) experimental method explores the pricing behavior when the monopoly is forced by rules and regulation to set prices that constrains the optimal choice of the monopoly.

The method of exploring the price behavior of the monopolist is unique compared to the literature introduced so far. Departing from Armstrong’s (2006) study he sets up four different pricing conditions (Nedelescu, 2013: 27) using the unrestrained monopolist as a benchmark model, the three following scenarios indicate 2. Prices to be set above cost 3. No price discrimination between the two sides and 4. High costs for the side valuing the platform less. Given the focus of our paper, only the outcome of the benchmark model is of interest. For each

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\(^{35}\) In a similar fashion of fulfilled expectations in equilibrium, unselective attention means no discrepancy between perceived software quality and actual software quality within their setup, in some ways you can argue that consumers are “perfectly informed” regarding the software quality of the other side.

\(^{36}\) The number of participants matter more than who these participants are.
setup Nedelscu calculates the theoretical predicted equilibrium prices, number of agents, profit and consumer surplus (Ibid: 30).

In order to test the actual pricing behavior of the monopolist, with the hypothesis that heterogeneous prices and some prices set below cost are profit maximizing and welfare enhancing, the experimental approach utilizes students acting as the monopolists setting prices.

The relevancy to the current study is not only in the concrete results of the dynamic pricing indicative of two-sided markets but we’ll find relevancy in the learning process of the students acting as monopolists, setting different prices to the two sides without knowing the demand structure of the agents. As it turns out, the students pricing behavior, without restriction, did not quite reach the profit maximizing theoretical counterpart, albeit a price below cost was pursued where beneficial. Some students neglected to set prices below cost at all (Ibid: 48).

3. The model

The foundation of the simulations presented in this paper is based on Gabszewicz & Wauthy (2005)\textsuperscript{37}. We initially aim at replicating their findings in an agent-based framework, and if successful, later use these results as a benchmark model to expand the model according to some of the suggestions made in their final remarks (Ibid: 10). This could include numerous avenues to travel through such as invoking heterogeneous agent behavior, introducing limited information with respect of the pricing behavior of the monopolist, adaptive expectations for the agents and perhaps use empirically observed stylized facts as assumptions to simulate more realistic outcomes. In length this will also provide some knowledge towards the usefulness of approaching economics as a complex adaptive system with the additional tools of computer simulations beside regression analysis and mathematical modeling.

GW outlays a simple framework of a two-sided market with visitors, exhibitors and a fair\textsuperscript{38} acting as the platform. They consider, model and derive equilibrium conditions for three types of competition settings, of which only the case of the monopoly will be considered.\textsuperscript{39}

(1) The monopoly case – Equilibrium prices, profits and number of active agents\textsuperscript{40} with one platform.

This paper intends to expand on the first theoretical model in two regards beside basic replication.

\textsuperscript{37} Henceforth abbreviated as GW  
\textsuperscript{38} Fair, exhibition centre and monopoly will all be used as synonyms in the paper.  
\textsuperscript{39} The two other models both deal with duopolies, one model with only single-homing, the other with multi-homing.  
\textsuperscript{40} Active agents is the term used to depict the number of agents that visit the fair
Initially we focus our attentions on the agents, the two sides aiming to visit the exhibition centre. As often assumed in the literature of networks, the expectations of agents are said to be fulfilled (Katz & Shapiro, 1985) at equilibrium. Albeit necessary to derive closed-form solutions in a theoretical model, working in a simulation framework we could alleviate this assumption in order to explore when the two sides aren’t coordinated.\textsuperscript{41} In length we are also directly taking up the suggestion made in GW final remarks (Ibid: 10 - 11) regarding the use of active agents.

In a second instant we will aim to alleviate the assumption of perfect information. To clarify, the assumption that the monopoly knows the demand structure of the two sides and is able to set the optimal prizes for profit maximization will be discarded for the benefit of insights from Simon (1955) and the concept of satisficing. Working under the relatively more realistic belief that the monopoly is not able to gather all the information necessary to set the profit maximizing prices we shall instead aim toward programming the pricing behaviour to loosely resemble how platforms empirically set their prices.

For an overview of which assumptions are used and when, table 1 can be of assistance. Exactly how the more active behaviours from the agents and how the monopoly will set its prices under limited information will be programmed is explained in the relevant subsection.

<table>
<thead>
<tr>
<th>Cases to be run</th>
<th>Monopoly-behaviour</th>
<th>Agent-behaviour</th>
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<td>Case 1</td>
<td>Perfect information</td>
<td>Fixed fulfilled expectations</td>
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<td>Case 2</td>
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<td>Case 3</td>
<td>Myopic pricing behaviour</td>
<td>Fixed fulfilled expectations</td>
</tr>
<tr>
<td>Case 4</td>
<td>Perfect information</td>
<td>Local expectations</td>
</tr>
</tbody>
</table>

Table 1 General overview of implemented provider and agent behavior

In all models we assume that there are 100 agents on each side of the platform, the total population is thus fixed to 100 visitors, 100 exhibitors and one platform provider. Given that the replication of the initial monopoly model by GW is successful, the resulting profits, prices and active participants will be benchmark variables in which latter simulations are compared with.

\textsuperscript{41} To my understanding in the theoretical literature there exists an equilibrium for each expected network size. The stringent assumption is that whatever we assume that the agents expect, it is essential for the equilibrium outcome that their expectations are fulfilled. Every agent/group is coordinated in their beliefs with everyone else.
The agents
Following GW (Ibid: 3) the model consists of three types of agents. Visitors, exhibitors and a commercial fair which serves as our platform aiming to attract the two sides.\textsuperscript{42}

Visitors
Visitors have a utility function linear in the number of exhibitors and the price of entering the fair:

1. \( U_v = \theta_i x - p_v \)

And where \( \theta \) is assumed to be uniformly distributed in a \([0, 1]\) interval. \( \theta \) can be seen to measure the importance and the number of transactions each visitor wish to perform (Ibid: 4). \( x \) is the actual number of active exhibitors and \( p_v \) reflects the price paid for entry.

The utility of visitors not participating is normalized to zero.

Exhibitors
Exhibitors have a utility function linear in the number of visitors and the price of entering the fair:

2. \( U_x = \gamma_i v - p_x \)

And where \( \gamma \) is assumed to be uniformly distributed in a \([0, 1]\) interval. \( \gamma \) can be seen to measure the value of the good exhibitors have for sale in the fair (Ibid: 3). \( v \) is the actual number of active visitors and \( p_x \) reflects the price paid for entry.

The utility of exhibitors not participating is normalized to zero.

Platform (Monopolist)
As indicated before, the monopolist seek to maximize profits through setting \( p_v \) and \( p_x \). We model the following demand functions which are the target objectives to be maximized:

3. \( D^v(p_v, x^e) = 1 - \frac{p_v}{x^e} \)

4. \( D^x(p_x, v^e) = 1 - \frac{p_x}{v^e} \)

5. \( p_x = \frac{v^e}{2} \)

6. \( p_v = \frac{x^e}{2} \)

7. \( \pi^* = p_vD^v(p_v, x^e) + p_xD^x(p_x, v^e) \)

\textsuperscript{42} A more elaborate description can be found in the original paper, due to the objective of this paper the author has only tried to keep the essential information necessary for implementing the model in a computer simulation.
For brevity a table summarizing the initial chosen parameters, their values and previously mentioned functions is supplied in the appendix, table 18.

4. The program and initialization
The chosen program for conducting the simulation is NetLogo. NetLogo is an agent-based modeling software used in social science. There are several different software when it comes to simulating ABMs the current choice is the outcome of several different factors: The authors own lack of programing experience with the promise of ease of use (Wilensky & Rand, 2015: xiv) the large library of models from previous works to draw inspiration of, the software being free as well as a seemingly relatively active community dealing with programing issues that might come up. The metaphor GW uses of a commercial fair is convenient in terms of the graphical aspect of implementing a simulation model.

Case 1: Perfect information and fixed fulfilled expectations - replicating Gabszewicz & Wauthy (2005).
In order to replicate the equilibrium outcome of their model in an agent-based framework some practical issues occur which has to be taken into consideration. The assumption of an infinite population is not one we can replicate, thus in order to mimic the uniform distribution of $\theta$ and $\gamma$ for our population of 100 agents on each side a grid-like distribution will be programmed as the solution. Agents are given their unique value of $\theta$ and $\gamma$ ranging from 0 – 1 with 0.01 increments.

| $\theta_{visitor1}$ = 0.01, $\theta_{visitor2}$ = 0.02, $\theta_{visitor3}$ = 0.03, $\theta_{visitor4}$ = 0.04... $\theta_{visitor100}$ = 1 |  
| $\gamma_{exhibitor1}$ = 0.01, $\gamma_{exhibitor2}$ = 0.02, $\gamma_{exhibitor3}$ = 0.03, $\gamma_{exhibitor4}$ = 0.04... $\gamma_{exhibitor100}$ = 1 |

Textbox 1: Uniform distribution with a finite population

The first model will be an attempt to replicate the monopoly outcome. Using the same assumptions as GW we should not expect any dynamic behaviour to be present at all. In GW the agents are assumed to have rational expectations with perfect information. The number of active agents are instantly aligned with their expectations of how many active agents that will be present on the other side, that is:

$8. x = x^e, 9. v = v^e$

The utility functions our agents act upon are:

$10. U_v = \theta x^e - p_v, 11. U_x = \gamma v^e - p_x$

---

43 Especially for someone not familiar with this type of method
44 Fixed fulfilled expectations is another term commonly used in the literature, which to my understanding has the same meaning.
There is thus instantaneous alignment between expected utility (eq. 10 and eq. 11) and realized utility (eq. 1 and eq. 2) under the assumption of fixed fulfilled expectations.

The monopolist sets prices as previously outlined in equation 5 and 6:

\[ 12. \quad p_x = \frac{v^e}{2}, \quad 13. \quad p_v = \frac{x^e}{2} \]

The importance of managing to replicate the initial model could be argued to be two fold. Given that all agents have perfect information and with fixed fulfilled expectations the static outcome will work as a benchmark model in which further models will be compared with. We are only aiming to conform the closed analytical solution in a broader framework in which further complexities can be explored. Using the optimal analytical solution of how the agents *should behave* (*Toulmin: 1992:125*) as a benchmark model avails us to compare exactly how different types of behaviours qualitatively and quantitatively differ from the behaviour with perfect information and static beliefs.

The second and arguably more important reason is that we add fractionally to the validity of further, relatively more involved models. If we know that at least the benchmark model is correctly replicated in a simulation setting, the foundation could be argued to be somewhat sound. The lack of overall validation and verification of forthcoming models in the paper is considered by the author to be an issue. How to validate, verify and/or replicate simulation studies seem to be an ongoing discussion (Huotari et. al., 2016:9) however some tools do seem to be utilized in order to satisfy this very important necessity in running experiments through simulations (Wilensky & Rand, 2015: Chapter 7). These tools and their lack of possible implementation in this current paper will be discussed in length in the later sections of this paper.

In the spirit of changing the model in increments in different directions instead of assuming that the population is uniformly distributed, we let \( \theta \) and \( \gamma \) be random draws from a uniform distribution in a second model. Each agent is attributed with a random draw from \( 0 - 1 \) in their \( \theta \) and \( \gamma \) values. The textbox below shows an example of how the values can be randomized

\[
\begin{align*}
\theta_{\text{visitor}1} &= 0.5423, \theta_{\text{visitor}2} = 0.412, \theta_{\text{visitor}3} = 0.963, \theta_{\text{visitor}4} = 0.326, \ldots \theta_{\text{visitor}100} = 0.235 \\
\gamma_{\text{exhibitor}1} &= 0.986, \gamma_{\text{exhibitor}2} = 0.875, \gamma_{\text{exhibitor}3} = 0.03, \gamma_{\text{exhibitor}4} = 0.23, \ldots \gamma_{\text{exhibitor}100} = 0.12
\end{align*}
\]

*Textbox 2: Uniform distribution with a random draw for a finite population*
In order to further clarify the differences of the two models that are related to case 1, table 2 outlays the similarities and differences of the assumptions made.

Questions we set out to answer with simulation one are thus the following:

1. Can we replicate the analytical solution in a computer simulation?
2. Do the qualitative predictions of the analytical solution hold for a random draw of \( \theta \) and \( \gamma \)?

**Case 1 – Stage game.**

In replicating G&H analytical solution, all three agents are coordinated in their expectations and behaviors. This is also how the game is implemented and coded in NetLogo. The stage game is as follows for case 1:

1. Agents have fixed fulfilled expectations regarding the expected attendance of the other side, hence their expectations equates to:

\[
\begin{align*}
\bar{x}_t &= x_t = 50 \\
\bar{v}_t &= v_t = 50
\end{align*}
\]

2. The monopolist also knowledgeable of the expectations of the agents sets the profit maximizing prices to be:

\[
\begin{align*}
p_{v,t} &= \frac{\bar{x}_t}{2} = 25, \\
p_{x,t} &= \frac{\bar{v}_t}{2} = 25
\end{align*}
\]

3. Agents act according to their expected utility function, which in current case corresponds to their actual utility function instantly as \( x_t = x_t^e \) and \( v_t = v_t^e \)

\[
\begin{align*}
U_{v,t} &= \theta_i x_t^e - p_{v,t} \\
U_{x,t} &= \gamma_i v_t^e - p_{x,t}
\end{align*}
\]

4. The monopolist receive:

\[
\pi_t = p_{v,t} * v_t + p_{x,t} * x_t
\]

Since all parties take advantage of the same and correct information there is instantaneous alignment. The only exhibitors and visitors that move according to step 3 are the ones who will remain on the platform. Thinking of this stage-game in an agent-based framework we would thus anticipate that the simulations converge instantly. Table 3 summarizes what we should expect by the first model.
In case 1 the decision is made to let the simulation run for 20 time periods. Manually testing the model for different lengths of time as well as running the model in a more systematic manner with longer and shorter time periods, given the quick convergence, 20 time periods deemed sufficient to ensure we have reached a stable point of no agents moving back and forth to and from the fair.

**Case 1.1**

In order to find the profit-maximizing prices when $\theta$ and $\gamma$ are randomly drawn we use a genetic algorithm. When $\theta$ and $\gamma$ are randomly drawn the price-setting behavior of step 2 explained for case 1 will no longer hold. We make use of the same model as case 1, where the agents move according to their utility functions however the code is changed in favour of the algorithm being able to manipulate the prices in order to maximize the monopolist profit function. The process is programmed as follows:

1. We specify the function that the algorithm should maximize, in this case:
   \[ \pi = p_v \cdot v + p_x \cdot x \]
2. We specify the prices it should iterate through, which in this case is set to:
   \[ p_v = [0 - 50], p_x = [0 - 50] \]
3. We specify in what increment, which in this case is set to:
   \[ p_v \pm 1, p_x \pm 1 \]

---

45 $x$ and $v$ are the corresponding active agents on each side under the assumption of fulfilled expectations.
46 From my understanding a genetic algorithm is a computerized search method slightly more effective in its searching behavior than a random search algorithm. The algorithm is implemented through a complementary tool called BehaviorSearch, developed for NetLogo. [http://www.behaviorsearch.org/](http://www.behaviorsearch.org/)
47 With an infinite population and uniformly distributed $\theta$ and $\gamma$ the platform will always entertain $\frac{1}{2}$ of the agents on both sides, when implementing $\theta$ and $\gamma$ to be randomly drawn with a finite population this does not necessarily have to be the case as the distribution of $\theta$ and $\gamma$ could invoke the number of visitors with $\theta > \frac{1}{2}$ to be more than 50 visitors and vice versa for exhibitors.
48 I am referring to the NetLogo model
4. Agents will move back and forth depending on their utility functions as previously stated

\[ U_{v,t} = \theta_i x_t^e - p_{v,t} \]
\[ U_{x,t} = \gamma_i v_t^e - p_{x,t} \]

Essentially the algorithm is an automated search process, our agents will move back and forth reacting to the price set by the algorithm and for each price change the algorithm calculates the profit in search of better combinations. How long the algorithm should search and for how many simulations is then up to the researcher and the computing abilities of the hardware used.

In our case 200 searches – simulations – were specified with the algorithm searching through 500 combinations of different prices.

**Case 2: Perfect information with naïve- and adaptive expectations**

Simulation 2 focuses our attention on making our agents slightly more complex. Exhibitors, visitors and the monopolist form expectations about the number of agents on the other side that will adopt the platform in a slightly more elaborate manner. We are thus relaxing the assumption used in the theoretical literature on network effects of fulfilled expectations\(^49\).

The rather simple adaptive behaviour implemented in this paper is derived from the naïve- and adaptive expectation hypothesis often used in Cobweb models (Velden: Chapter 2, Hommes: 1994).

\[ x^e = \lambda x_{t-1} + (1 - \lambda)x_{t-2} \]
\[ v^e = \lambda v_{t-1} + (1 - \lambda)v_{t-2} \]

All agents on either sides are homogenous in their expectations, thus all visitors expect the same number of exhibitors and vice versa. However the expectations of the two sides can differ.

Whenever \( \lambda = 1 \) the expectations of our agents becomes \( x^e = x_{t-1}, v^e = v_{t-1} \) hence they are considered naïve as they expect the number of active agents of the other side to conform to what was observed the previous period. Being able to modify \( 0 \leq \lambda \leq 1 \) allows us to simulate outcomes under different adjustment values between the two previous periods. This is where the literature starts to consider the agents being “adaptive”, moving somewhat away from being naïve (Hommes, 1998: 338 - 344).

The utility functions our agents act upon are changed from case 1 (eq. 10 & eq. 11) to:

\[ U_v = \theta(\lambda x_{t-1} + (1 - \lambda)x_{t-2}) - p_v \]
\[ U_x = \gamma(\lambda v_{t-1} + (1 - \lambda)v_{t-2}) - p_x \]

In a 2nd step, given the choice they’ve made, they evaluate their realized utility function (eq. 1 & eq. 2), depending on the actual active number of agents on the other side.

The monopoly sets prices based on the adaptive expectations of the agents, simply using the right hand side of equations (14) and (15) into the price function, the monopolist sets the profit maximizing prices to be:

18. \[ p_v = \frac{(\lambda x_{t-1} + (1 - \lambda)x_{t-2})}{2} \]
19. \[ p_x = \frac{(\lambda v_{t-1} + (1 - \lambda)v_{t-2})}{2} \]

The practical issue of implementing adaptive behaviour is in our case how to initialize the previous periods expected network sizes \( x_{t-1}, x_{t-2} \) and \( v_{t-1}, v_{t-2} \) before anything has yet occurred. Using standard procedure we will let NetLogo randomize different starting points in each simulation. The strength of randomizing the initialized values lies in checking the robustness of convergence and to find potential different basins of attractions.

<table>
<thead>
<tr>
<th>Cases to be run</th>
<th>Monopoly-behaviour</th>
<th>Agent-behaviour</th>
<th>Assumption of ( \theta ) and ( \gamma )</th>
<th>Initialized expectations ( x_{t-1}, x_{t-2} )</th>
<th>Initialized expectations ( v_{t-1}, v_{t-2} )</th>
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</thead>
<tbody>
<tr>
<td>Case 2.1.</td>
<td>Perfect information</td>
<td>Naïve expectations</td>
<td>Uniform distribution</td>
<td>Random value 0 - 100</td>
<td>Random value 0 - 100</td>
</tr>
<tr>
<td>Case 2.2.</td>
<td>Perfect information</td>
<td>Adaptive expectations</td>
<td>Uniform distribution</td>
<td>Random value 0 – 100</td>
<td>Random value 0 – 100</td>
</tr>
</tbody>
</table>

Table 4 Assumptions used in the simulations regarding case 2.

Questions we set out to answer with simulation two are thus the following:

1. What are the outcomes when agents have naïve expectations?
2. What are the outcomes when agents have adaptive expectations?

Case 2 – Stage game.

The stage-game can be depicted as follows, this structure follows for both #2.1 and #2.2 with the difference of the latter having \( \lambda \) playing an active role.

1. Agents form expectations according to equation 14 and 15:
   \[ x_t^e = \lambda x_{t-1} + (1 - \lambda)x_{t-2}, \quad v_t^e = \lambda v_{t-1} + (1 - \lambda)v_{t-2} \]
2. The monopoly observes the two most recent periods and sets the prices according to equation 18 and 19:
   \[ P_{x,t} = \frac{v_t^e}{2}, \quad P_{v,t} = \frac{x_t^e}{2} \]
3. Agents decide to adopt the platform according to their expected utility, that is if their expected utility is strictly positive $U_t > 0$

$$U_{v,t} = \theta_i (\lambda x_{t-1} + (1 - \lambda)x_{t-2}) - p_{v,t}$$

$$U_{x,t} = \gamma_i (\lambda v_{t-1} + (1 - \lambda)v_{t-2}) - p_{x,t}$$

4. Agents who chose to adopt the platform evaluate their realized utility (eq.1 and eq.2). If the utility is $U_t \leq 0$ they leave the platform

$$U_{x,t} = \theta_i * v_t - p_{x,t}$$

$$U_{v,t} = \gamma_i * x_t - p_{v,t}$$

5. The monopolist receives:

$$\pi_t = p_{v,t} * v_t + p_{x,t} * x_t$$

At the start of the simulation, we create two lists of randomized starting values representing the initial expectations of our agents, using random draws with values between 0 - 100.$^{50}$

$$[x_0, x_{-1}, x_{-2}]$$

$$[v_0, v_{-1}, v_{-2}]$$

Having established a “history” of previous expectations the agents form their expectations regarding the number of active agents on the other side in step 1. Step 2 determines the prices that our monopolist sets. In cases related to 2.1 and 2.2 the price-setting behaviour is done according to the behaviour stated previously, of our agents being backward-looking in their belief formations and the monopolist having knowledge of this fact.

In step 3 we implement the decision of whether our agents conform to the platform or not, this is when they both consider their own expectations as well as the price they now observe set by the monopolist.

Step 1 – 3 can be thought of as actions taken by our agents before the fair opens, the monopolist has set its prices for the day and the agents have made up their mind based on their expectations and the observed price.

In step 4 the fair opens and our agents move according to the decision they’ve made by the equations in step 3. Intuitively one can think of step 4 as the agents reaching the fair and reassessing their utility according to the number of agents on the other side that they see. If the number of agents corresponds to their utility being positive, they pay the price of entry otherwise.

---

$^{50}$ This has to be done first in NetLogo because if the agents formulate their beliefs before coding the pricing-behavior they will not consider the price when evaluating their expected utility.
they leave. At the end of the day, step 5, the monopolist closes the fair and calculates the days’ profits.

The list of previous expectations is updated with the number of active participants on both sides of the market before a new day begins. The steps from 1 - 5 are then iterated until we decide to stop the simulation.

In case 2 the decision is made to let the simulation run for 20 time periods under the same justification made previously for case 1. Even though the simulation seem to require a few more steps in some instances 20 steps is still a wide enough margin to ensure that the simulation has settled.

**Case 3: Imperfect information, satisficing and fixed expectations**

In this simulation we are ready to relax the assumption of perfect information for the monopolist. How complex one could make the pricing decision with uncertainty and unknown demand structures can be argued to depend on several factors, i.e. programing experience on part of the researcher, gathered empirical evidence on pricing decisions, theoretical guidance, using regressions to model for hedonic pricing behaviour etc. With a specific market in mind and relevant empirical data gathered one can thus make the pricing decision more accurate than we are able to in the current paper. The satisficing behaviour we utilize in this paper is similar to step two of the provider decision regarding reinforcement learning in the paper by Heinreich and Gräbner (2015: 15).

The monopoly initiates pricing endogenously at very low rates starting with

\[
20. p_v = 1, \quad 21. p_x = 1
\]

and gradually increases prices until the point where the profit of preceding period exceeds current profits, this is the 2nd step of Ibid (2015).53

\[
\begin{align*}
    &\text{if } \pi_{x,t} > \pi_{x,t_0} \rightarrow p_{x,t} = p_{x,t-1} + 1 \\
    &\text{if } \pi_{x,t} < \pi_{x,t_0} \rightarrow p_{x,t} = p_{x,t-1} \\
    &\text{if } \pi_{v,t} > \pi_{v,t_0} \rightarrow p_{v,t} = p_{v,t-1} + 1 \\
    &\text{if } \pi_{v,t} < \pi_{v,t_0} \rightarrow p_{v,t} = p_{v,t-1}
\end{align*}
\]

When this threshold is reached we argue that the monopoly has reached its satisficing condition. The behaviour is thus similar to the adaptive agents in the previous case, the monopolist is

---

51 In the way of coding this behavior the implementation is obviously less realistic. In our simulations the agents move back and forth each time period until we reach some stabilized state of no agents moving away or to the platform, hence they do not stop at the “door” as depicted in the story here however the outcome remains the same.

52 Steps, tick/s and time period will be used interchangeably.

53 I would like to add however that ours is much more simplistic in nature.
myopic in the sense that it only observes, remembers and compares its previous profits of making decisions considering price.

Pricing low\textsuperscript{54} to build a network of agents on both sides is how most digital platforms operate. Evans (2003) provide an overview of different well explored two sided markets such as the credit card industry (Ibid: 201 – 203) and gaming console industry (Ibid: 195 - 196) providing (selling) credit cards (the hardware) initially at a loss in order to increase the number of holders (gamers). Increasing prices or incorporating less favourable conditions with time and as the platform manages to grow in sufficient size seem to be common practice. Evidence\textsuperscript{55} of the latter claim could be found in the increasing dissatisfaction of sellers on a platform such as in the online travel industry or restaurant take-away industry.\textsuperscript{56} Additionally, one could also think of less favourable conditions being implemented on the consumers through the increased usage of advertising on the platform or limiting services in favour of paying customers.\textsuperscript{57} Thus setting a very low initial price with gradual increases as the network grows is the argument made in the current case to justify the current pricing behaviour.\textsuperscript{58}

We again revert our agents to the same belief formation established in case 1, since we only want to make small departures from the rationality of GW in different directions. This will allow us to compare the myopic pricing behaviour of case 3 with that of case 1, to see how the myopic monopolist profits measures against the benchmark model. The agents are thus always assumed to expect 50 agents on the other side, \( x = x^e = 50, v = v^e = 50 \)

The utility functions our agents act upon are:

\begin{align*}
22. U_v &= \theta x^e - p_v \\
23. U_x &= \gamma v^e - p_x
\end{align*}

Besides making the monopolist myopic we will again let \( \theta \) and \( \gamma \) be random draws from the uniform distribution between 0 - 1 to see if this qualitatively changes potential outcomes.

<table>
<thead>
<tr>
<th>Cases to be run</th>
<th>Monopoly-behaviour</th>
<th>Agent-behaviour</th>
<th>Assumption of ( \theta ) and ( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 3. Q1</td>
<td>Myopic pricing behaviour</td>
<td>Fixed expectations</td>
<td>Uniform distribution</td>
</tr>
<tr>
<td>Case 3.1. Q2</td>
<td>Myopic pricing behaviour</td>
<td>Fixed expectations</td>
<td>Uniform distribution with a random draw</td>
</tr>
</tbody>
</table>

*Table 5 Assumptions used in the simulations regarding case 3.*

\textsuperscript{54} More often than not, subsidizing one side through negative prices. This was, for example, the early strategy of PayPal trying to grow: \url{https://www.referralcandy.com/blog/paypal-referrals/}

\textsuperscript{55} Or perhaps as “stylized-facts”

\textsuperscript{56} \url{http://www.gp.se/nyheter/ekonomi/44-pizzerior-rasar-mot-onlinepizza-1.172238}
\url{http://www.konkurrensverket.se/nyheter/utredning-ledde-till-fortydligande-av-onlinepizzas-alert/}
\url{http://www.dagensjuridik.se/2014/12/kkv-gar-till-botton-med-onlineresebyraer-risikerar-leda-till-otillaten-konkurrensbegransning}

\textsuperscript{57} This would be the often used freemium models of “payment” which can be upgraded for a small fee in order to access additional content and so forth.

\textsuperscript{58} In this respect we adhere to the calls of critiques arguing we make use of assumptions observed empirically, rather than detached theoretical assumptions.
Questions we set out to answer with simulation three are thus the following:

1. What are the market outcomes with a myopic monopoly pricing behaviour and agents with fulfilled expectations?
2. How do the results compare for a random draw of $\theta$ and $\gamma$?

Case 3 – Stage game.

The stage-game can be depicted as follows, this structure follows for both 3 and 3.1 with the difference of the latter we assume $\theta$ and $\gamma$ being a random draw between 0 – 1.

1. Agents have fixed expectations regarding attendance of the other side, hence their expectations are:

   $x_t^e = 50$
   $v_t^e = 50$

2. The monopolist initiates prices for the two agent groups at:

   $p_{v,t} = 1, p_{x,t} = 1$

3. Agents act according to their expected utility function, which in current case is fixed to their actual utility function as $x_t^e = x = 50, v_t^e = v = 50$:

   $U_{v,t} = \theta_i x_t^e - p_{v,t}$
   $U_{x,t} = \gamma_i v_t^e - p_{x,t}$

4. The monopolist receives:

   $\pi_{x,t} = p_{x,t} * x_t$
   $\pi_{v,t} = p_{v,t} * v_t$
   $\pi_t = p_{v,t} * v_t + p_{x,t} * x_t$

5. The monopoly updates prices:

   $\text{if } \pi_{x,t} > \pi_{x,t-1} \rightarrow p_{x,t} = p_{x,t-1} + 1$
   $\text{if } \pi_{x,t} < \pi_{x,t-1} \rightarrow p_{x,t} = p_{x,t-1}$
   $\text{if } \pi_{v,t} > \pi_{v,t-1} \rightarrow p_{v,t} = p_{v,t-1} + 1$
   $\text{if } \pi_{v,t} < \pi_{v,t-1} \rightarrow p_{v,t} = p_{v,t-1}$

At the start of the simulation we create three lists of previous profits for the monopolist, one for each agent group and a third summarizing both.\(^{59}\)

$[\pi_{x,0}, \pi_{x,-1}, \pi_{x,-2} \ldots]$
$[\pi_{v,0}, \pi_{v,-1}, \pi_{v,-2} \ldots]$
$[\pi_0, \pi_{-1}, \pi_{-2} \ldots]$

\(^{59}\) The profit without any specific subscript depicts the sum of $\pi_x + \pi_v = \pi$
This is required in the same way we required some starting point in expectations for case 2. The pricing behaviour requires some previous profit to be compared to. The starting values depicting previous profits in step 1 are kept fairly low, being a random number between 0 – 100.\textsuperscript{60}

In step 1 agents form their expectations. As in case 1 they always expect 50 active participants of the opposite side. In step 2 the monopolist sets the prices at 1 for both exhibitors and visitors under the justification made previously in this section.

Step 1 – 2 can be thought of as actions taken by our agents before the fair opens, the monopolist has set its prices for the day and the agents have made up their mind based on their expectations and the observed price. In step 3 the agents adopt the platform if their utility is strictly positive and in step 4 the monopolist calculates its profit. In step 5 the monopolist compares its profits with the previous period and updates prices accordingly.

All three lists are updated with the latest profits which the next periods profit will be compared to before a new day begins. The steps from 3 - 5 are then iterated until we decide to stop the simulation.

Manually testing the models for different lengths of time as well as running the model in a more systematic manner with longer and shorter time periods, 50 time periods was settled as sufficient to ensure that the simulation stabilizes for case 3, Q1 and 40 time periods was settled for case 3.1 Q2.

**Case 4 Intra-heterogeneity in expectations and local interaction**

In addition to heterogeneity between the two sides in their expectations we can introduce heterogeneity by letting each agent to have their own unique expectations. In a similar manner as established in case 2 the starting points in previous expectations before anything has yet occurred is a random value between 0 – 100 unique to each individual in our population.

We make use of the selective attention assumption of Huotari et.al (2016)\textsuperscript{61}. In our context, the simulations 2.1 and 2.2 can be argued to have used a global perspective. Each agent have been updated with the correct number of active participants of the other side disregarding on where they stand on the platform or how many visitors- and exhibitors are in close proximity.\textsuperscript{62} In order to introduce bounded behaviour in belief formations on part of our agents we program their behaviour to only perceive and update their beliefs by the number of agents on the other side they can see in their “neighbourhood”.\textsuperscript{63} Likewise we use the same principal in establishing their utility to be a function of their local neighbourhood, thus the name local interaction. With a

\textsuperscript{60} This is to ensure that the monopolist actually raises prices and the simulation gets started.

\textsuperscript{61} To iterate, in their paper they argue for selective attention to pose potential discrepancy between perceived and actual quality of complementary software, and that we might be biased in how we evaluate complementary software depending on how new the software is (release date).

\textsuperscript{62} This is what we have used in the naïve- and adaptive belief formations so far.

\textsuperscript{63} We should not interpret this behavior beyond any theoretical exploration of how our simulation evolves when agents update their beliefs in a bounded way instead of the previous “global” cases.
limited range of how far they are allowed to see their utility will essentially generally be lower than the previous cases considered so far.

What would make this case interesting is not only the increased heterogeneity of the initializing conditions but the continued stochastic dynamic formation of agents’ beliefs depending on their type\(^{64}\), their position in our “world” and the number of agents of the other side in the vicinity of their programmed vision. An additional feature is that as the agents move around they do so in a manner resembling a random walk, whether they are present on the platform or not they are in continuous movement. An example of how the space looks like can be found in the appendix, figure 12. The blue patch in the middle of our world represents the fair, and the white patches can be considered the area for those agents not finding it beneficial to opt in to the fair.

Introducing local interaction in this way we are departing from \(\pi = 2500\) being the equilibrium outcome. The additional heterogeneity and the stochastic local interaction will probably induce the market outcome to bounce around, question is if it stabilizes at a certain distribution.

Visitors form their expectations through equation 24:

\[
24. x_f^e = \text{exhibitors in cone distance}_{i,t-1} = x_{i,t-1}
\]

In-cone is the NetLogo primitive\(^{65}\) used to establish how many exhibitors they see in their local environment and \(\text{distance}\) is the argument of how far we allow them to see. Distance can take values from 0 – 20, it will play the same role that \(\lambda\) previously had. Changing the distance, \(\text{distance} \to 20\) we expect the agents to see further. In the simulations to come the local interaction will only be considered for visitors. I will only allow the simulations to change the distance of the visitors, and the values considered will be 0 – 16\(^{66}\). We still assume the visitors being naïve in the same way as case 2 with the difference of locally updated expectations.

The expectations of exhibitors is set by:

\[
25. v^e = v_{t-1}
\]

Hence they maintain the naïve belief formations of case 2\(^{67}\). This is again in light of only making small departures in different directions to see how the model develops.

The utility functions our agents act upon are again based on the previous period:

\[
26. U_v = \theta x_{i,t-1} - p_v \\
27. U_x = \gamma v_{t-1} - p_x
\]

---

\(^{64}\) Their \(\theta\) and \(\gamma\)

\(^{65}\) Primitives are built-in functions of NetLogo

\(^{66}\) The collected data became too vast to handle for the programs and computer in use of the simulations if we were to stretch distance any further, hence this cut off.

\(^{67}\) Note that although each exhibitor is given a unique list of previous expectations as a starting point, when the simulation starts all their expectations will converge to the same number because they take advantage of the same “global” information handed to them.
Granted that they have acted according to their expected utility, the ones who adopt the platform evaluate their realized utility depending on the actual number of active agents of the other side. This is made locally for visitors and is kept global for exhibitors (eq. 2, which is the same as eq. 29).

\[
28. U_v = \theta (\text{exhibitors in-cone distance}) - p_v
\]

\[
29. U_x = \gamma v - p_x
\]

The monopolist will still maintain the pricing decision as a function of expectations, however as each one of our 100 agents have different expectations for each previous period the monopolist will settle for an average number of agents that met in the previous period, it is not able to price discriminate.

\[
30. \text{set } P_v = \frac{\text{mean } x_{i,t-1}}{2}
\]

\[
31. \text{set } P_x = \frac{\text{mean } v_{t-1}}{2}
\]

<table>
<thead>
<tr>
<th>Cases to be run</th>
<th>Monopoly-behaviour</th>
<th>Visitor-behaviour</th>
<th>Exhibitor-behaviour</th>
<th>Assumption of $\theta$ and $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 4, Q1</td>
<td>Perfect information</td>
<td>Naïve expectations</td>
<td>Naïve expectations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Locally updated</td>
<td>Globally updated</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Heterogeneous updated beliefs)</td>
<td>(Homogenous updated beliefs)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Uniform distribution</td>
<td></td>
</tr>
</tbody>
</table>

Table 6 Assumptions used in the simulations regarding case 4.

Questions we set out to answer:

1. What are the market outcomes when visitors are bounded by local expectations?

Case 4 – Stage game

The stage-game can be depicted as follows

1. Agents form expectations according to equations 24 and 25:

\[
x_{i,t}^e = \text{exhibitors in-cone distance}_{i,t-1}, v_{i,t}^e = v_{t-1}
\]

2. The monopoly observes the most recent periods and sets the prices according to equation 30 and 31:
\[ P_{v,t} = \frac{(\text{mean } x_{i,t}^e)}{2}, P_{x,t} = \frac{(\text{mean } v_{i,t}^e)}{2} \]

3. Agents decide to adopt the platform according to their expected utility, that is if their expected utility is strictly positive \( U_t > 0 \)

\[ U_{v,t} = \theta_i x_{i,t}^e - p_{v,t} \]
\[ U_{x,t} = \gamma_i v_{i,t}^e - p_{x,t} \]

4. Agents who chose to adopt the platform evaluate their realized utility (eq. 28 and eq. 29). If the utility is \( U_t \leq 0 \) they leave the platform

\[ U_{v,t} = \theta_i \ast \left( \text{exhibitors in– cone distance}_i,t \right) - p_{v,t} \]
\[ U_{x} = \gamma_i \ast v_{t} - p_{x,t} \]

a. Visitors who choose to adopt the platform continuously revise their utility according to the random walk feature and how many exhibitors they see.

b. The monopolist continuously revise prices respectively

5. The monopolist receives:

\[ \pi_t = p_{v,t} \ast v_{t} + p_{x,t} \ast x_{t} \]

We create two lists of randomized starting values representing the previous expectations of our agents. The initialized values are now unique for each agent.

\[ [x_{i,0}, x_{i,-1}, x_{i,-2} \ldots ] \]
\[ [v_{i,0}, v_{i,-1}, v_{i,-2} \ldots ] \]

Having established a “history” of previous expectations the agents form their expectations regarding the number of active agents on the other side in step 1. In step 2 the monopolist sets the prices reflecting the average number of agents who met in the previous period. When initializing this setup taking the average will be necessary for both agent groups as exhibitors are initially assumed to have their own unique “take” of what the number of active visitors were last period as well. Reaching the point where the simulation starts to iterate exhibitors make use of the same information and the expectations will become homogenous.

---

Note the \( i \) subscript distinguishing case 4 with case 2
In step 3 we implement the decision of whether our agents conform to the platform or not, this is when they both consider their own expectations as well as the price they now observe set by the monopolist.

Step 1 – 3 can be thought of as actions taken by our agents before the fair opens, the monopolist has set its prices for the day and the agents have made up their mind based on their expectations and the observed price.

In step 4 the fair opens and our agents move according to the decision they’ve made by the equations in step 3. Intuitively one can think of step 4 as the agents reaching the fair and reassessing their utility according to the number of agents on the other side that they see. 4a and 4b can be seen as revisions done continuously for the agents and monopolist. Reaching step 5 and the profit of the monopolist, it is less certain how and when this step can be consider to be fulfilled in light of 4a and 4b. In some sense the monopolist continuously observes its profits while the list of previous expectations are revised in this continuous motion as well.

The list of previous expectations is updated with the number of active participants on both sides of the market before a new day begins. The steps from 1 - 5 are then iterated until we decide to stop the simulation. In this case we settle for 100 time periods and 500 simulations for each vision setting from \([0 – 16]\). After 100 time periods the simulation seemed to have settled at a certain distribution, but the main reason for 500 simulations and 100 time periods for each vision setting comes down to data management issues, in some aspect we weigh in the importance of running enough models for enough time periods.

5. Results

Case #1. Replicating the monopoly equilibrium. Q1.
The replication of the analytical approach, given our numerical values, in an agent based framework seem to hold. Running the simulation 5000 times for reassurance the simulations depicts the static outcomes derived in table 2\(^69\). Each simulation was given 20 time periods to settle.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>5000</td>
<td>2500</td>
<td>0</td>
<td>2500</td>
<td>2500</td>
</tr>
<tr>
<td>Active visitors</td>
<td>5000</td>
<td>50</td>
<td>0</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Active exhibitors</td>
<td>5000</td>
<td>50</td>
<td>0</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Price visitors</td>
<td>5000</td>
<td>25</td>
<td>0</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Price exhibitors</td>
<td>5000</td>
<td>25</td>
<td>0</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 7 Summary statistics for the last step of each observation

\(^{69}\) Beside the fact that we manage to replicate the equilibrium outcome of proposition 1 under standard neoclassical assumptions the author can also conclude that the work and effort spent programming case 1 far exceeds the value of the results produced, hence in this case the analytical solution is perhaps more superior than the simulation approach.
The graphical output of a typical simulation is shown in the appendix, figure 11. In short we are successful in replicating proposition 1 of the analytical solution in an agent-based setting. An alternative distribution of agents and prices that exists according to GW that are profit-maximizing as well would be for the monopolist to give away the entry for free on one side whilst charging the other side double the interior equilibrium price presented in table 3. We are not successful in replicating proposition 2 in their paper (Ibid: 5). Each comparison to the benchmark equilibrium for subsequent models to come will henceforth be made with proposition 1 in mind.

Case #1.1. Equilibrium outcome with a random draw of $\theta$ and $\gamma$. Q2.

To test the generality of GW analytical solution by using a random draw for $\theta$ and $\gamma$ we take advantage of the genetic algorithm explained in the setup section previously.  

Depicted in table 8, we ran the model 200 times. By the results obtained one could argue that the distribution of $\theta$ and $\gamma$ provided by the randomness of NetLogo to be reasonably regular, which is a condition GW highlights as important for their results to hold (Ibid: 10). The algorithm comes very close to the analytical solution of proposition 1 with an infinite population, as can be seen by figure 1.

What is more interesting however is that by taking a random draw from the uniform distribution rather than having an infinite population the set of optimal prices for a monopoly with perfect information is broadened. With a discrete draw the equilibrium will not be the same every time, each simulation has its own unique equilibrium conditions depending on how $\theta$ and $\gamma$ are distributed. By invoking this change in $\theta$ and $\gamma$ we open up for inter- and intra-heterogeneity, the active agents within any given side of the market as well as the optimal distribution of active agents between the two sides will be different compared to the theoretical counterpart of a uniform distribution.

---

70 Although our established pricing behavior (eq.5. and eq.6.) comes quite close, further investigation revealed that it did not quite reach the profit maximizing pair of prices, hence the use of the algorithm.

71 The process took about eight hours.
and an infinite population. To make matters concrete, as the benchmark model forces equilibrium values to what we presented in table 7, the number of active agents that represents the profit maximizing conditions in the cases of $\theta$ and $\gamma$ being randomly drawn may very well be, for example, $> 50$ for visitors and in the same instant be $< 50$ for exhibitors.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>200</td>
<td>2498.93</td>
<td>49.75</td>
<td>2367.2</td>
<td>2658.2</td>
</tr>
<tr>
<td>Price visitors</td>
<td>200</td>
<td>25.47</td>
<td>3.58</td>
<td>17</td>
<td>34</td>
</tr>
<tr>
<td>Price exhibitors</td>
<td>200</td>
<td>25.07</td>
<td>3.54</td>
<td>16</td>
<td>34</td>
</tr>
</tbody>
</table>

*Table 8 Summary statistics of maximum profit and profit-maximizing prices with a random draw of $\theta$ and $\gamma$*

Figure 2 shows the optimal found prices for the corresponding highest profit for each search, given that the algorithm has functioned and managed to successfully pinpoint the profit maximizing prices each time. The figure is telling in that all the outcomes where, for example, the maximum profit settles at $2400 < \pi < 2500$ correspondingly is reached through multiple different set of prices represented by the colour teal.

![Figure 2 set of prices found toward visitors and exhibitors maximizing profits](image)

Although no real definition of what “qualitative” means with regard to the generality of GW analytical solution, the random draw with a finite population instead of an infinite population

---

72 The reason why we have decimal values when the prices iterated are whole integers comes down to the algorithm running each combination of prices a specified amount of time in a given search for different models and averaging these to one value. The different models produces different results because of the randomized $\theta$ and $\gamma$
along with our interpretation of the results indicates that the analytical solution derived for a uniform distribution is at least fairly robust when changed to a random draw between $0 – 1$. More interesting have been the increasing number of outcomes and prices present.

Case 2

#2.1 Naive expectations

Simulating the model with agents initialized as backward looking for one period two different outcomes are observed at the end of our simulations. We run the model 10 000 times for 20 periods each.

<table>
<thead>
<tr>
<th>$\pi$ at $t_{20}$</th>
<th>#2.1 Lambda = 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq.</td>
</tr>
<tr>
<td>1250</td>
<td>194</td>
</tr>
<tr>
<td>2500</td>
<td>9 806</td>
</tr>
<tr>
<td>Total</td>
<td>10 000</td>
</tr>
</tbody>
</table>

Table 9 Equilibrium outcomes in profits

By making the agents active in their formation of expectations a simple bifurcation seem to have evolved, leading to the possibility of two different equilibria. A closer look into these 194 cases one common denominator seem to be present for each case. Here the stochastic nature of how we programmed the history of previous expectations seem to play a small role. Whenever the initial conditions are that of at least one agent group expecting zero agents when the simulation starts leads to the monopolist not managing to align the two groups toward the benchmark equilibrium. Instead a 2-type cycle seem to develop with each step of the simulation, 50 agents of one side always being present on the platform while none from the other side. $^{73}$

![Active agents at each step, profit = 1250](image)

Figure 3 the 2-type cycle discoordination between the two agent groups

$^{73}$ Given the presence of the alternative simulation equilibrium of $\pi = 1250$ I chose to run #2.1 with starting positions of $v_{t-1} = 0$, and $x_{t-1} = 0$ to see whether this introduces an additional simulation equilibrium, correctly so under these conditions $\pi = 0$ is an outcome in the framework of potential outcomes.
The two agent groups are thus misaligned in their expectations, one group expecting 50 and the other expecting 0 for the period to come, which is the information they act upon. This then continues indefinitely with every other period one group expecting 50 and the other expecting 0.

The alternative equilibrium depicted here could be contrasted with proposition 2 in GW (2005: 5). We are not successful in replicating proposition 2 in their paper, but with naïve agents get this alteration instead.

Assuming our interpretation of the workings of the simulations are correct, weak indications of the importance of initial conditions are present.

In order to check for global convergence it would be interesting to see the variation of initialized values for the expectations before the simulation has begun, considering that 98% of our simulation outcomes manages to reach the benchmark equilibrium in this particular draw.

Figure 4 depicts the different combinations of expectations initialized at the simulation start. What the figure also tells us are that our model is rather robust in reaching the equilibrium outcome.

The only source of sensitivity causing different outcomes seem to be, thus far, any combination of zero being introduced at the start of the simulations. In slightly more technical terms: when our population is finite there is a nonzero probability of drawing initial expectations to be zero for either/both visitors and exhibitors. Otherwise all our results seem to be robust for the equilibrium of the benchmark model, even with naïve-expectations on part of our agents under different initialized settings.

---

74 As a sensitivity check on whether “zero” causes these alternative outcomes we have also tried the same simulations when 0 is not part of their beliefs, that is, the randomized starting expectations are \([1 - 100]\). Whenever this is done for #2.1, the \(\pi_0\) outcome and \(\pi_{1250}\) cycles seem to disappear.
#2.2 Adaptive expectations

Following the Cobweb literature by introducing $\lambda$ we slightly increase the “memory” of our agents by varying degree, depending on its value. As always so far, we are interested if the model converges to stable outcomes and to what values. The observant reader may remember that when $\lambda = 1$ we have naïve expectations, and thus should expect the same outcomes as #2.1. The following, table 11, is a table for each $\lambda$ between $[0-1]$, in increments of 0.1 in each simulation setting. We run each simulation 1 000 times, amounting to 11 000 simulations in total. Two simulation outcomes are observed when the simulations can be said to have stabilized, this condition have been set to 20 “steps/time periods”.

<table>
<thead>
<tr>
<th>$\pi_{t20}$</th>
<th>Corresponding prices and active agents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_v$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1250</td>
<td>$25_t, 0_{t+1}$</td>
</tr>
<tr>
<td>2500</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 10 Prices, profits and active agents observed so far for #2.1

<table>
<thead>
<tr>
<th>$\pi$ at $t_{20}$</th>
<th>#2.4 Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>2500</td>
<td>986</td>
</tr>
<tr>
<td>Cumulative N</td>
<td>1 000</td>
</tr>
</tbody>
</table>

Table 11 Equilibrium outcomes in profits for different values of $\lambda$

For $0 < \lambda < 1$ the benchmark equilibrium seem to be more robust compared to when $\lambda = 0$ or $\lambda = 1$. This could be explained by the only source of sensitivity induced in the underlying model, leading to the 0 profit, being less likely to be drawn. In order for the 0 profit to emerge for $0 < \lambda < 1$ this would require the starting positions of both previous time periods considered by the agents to equal 0, considering then that our agents are now using two previous periods in forming their beliefs any stochastically induced zero at the beginning of the simulation has less bearing on their behaviour. Auxiliary evidence would be that whenever $\lambda = 0$ or $\lambda = 1$ only one period is considered, and as such a single stochastically induced zero expectations in the initializing condition leads to the additional outcome of zero profits observed.

---

75 The story would be that they now consider the weighted average of the two previous periods, instead of just using the previous period when forming their expectations.

76 In order to not clutter the naïve- and adaptive scenarios and enhance the verification of our simulations they have been programmed separately hence this distinction in the text.
A closer look into the 14 cases where \( \lambda = 0 \) and \( \pi = 0 \) reveals a similar 2-type cycle presented for #2.1 (figure 3), except that in every other step of the simulation results in the benchmark equilibrium outcome instead of \( \pi = 1250 \). This is due to the fact that both sides’ expectations have been segmented in coordination. When \( \lambda = 0 \) our agents only act according to their expectations in \( t_{-2} \), which fluctuates between 0 and 50 for each period leading to this outcome.

The first row presents the simulation in question, the time period, current prices, active participants and profit. The second and third row are the expectations which we have previously established are what the agents base their decision on, to attend the fair or not, for previous periods and last row shows the profit for the different time periods.

When it comes to \( \lambda = 1 \) and \( \pi = 0 \), for each of the 22 cases any stochastically induced zero in the starting values leads to the network breaking down, leaving the monopolist with zero profits however.

Tying our results to the questions at hand for this section, up to this point the benchmark model equilibrium of proposition 1 seem stable when considering both changing the uniform distribution of \( \theta \) and \( \gamma \) to random draws (Case 1) as well as with some simple active beliefs under different initialized conditions on part of our agents (Case 2). We will now move away from the previous literature of Cobweb dynamics and try our hands on approaching the outcomes with a bounded pricing behavior.
Case 3

Case #3. Q1. Myopic monopolist and uniform distribution of $\theta$ and $\gamma$

Running case 3 for 5000 simulations under the same assumptions used by our first model (case #1) a myopic monopolist leads us to the benchmark equilibrium in every simulation.

<table>
<thead>
<tr>
<th>Run$^{79}$</th>
<th>Step$^{80}$: 28</th>
<th>$P_v = 26$</th>
<th>$P_x = 26$</th>
<th>$x = 48$</th>
<th>$v = 48$</th>
<th>$\pi = 2496$</th>
</tr>
</thead>
<tbody>
<tr>
<td>History profits visitors</td>
<td>$[1248_{t=28} \ 1248_{t=27} \ 1250_{t=26} \ 1248_{t=25} \ 1242_{t=24} \ 1232_{t=23} \ 1218_{t=22} \ 1200_{t=21} \ ...]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>History profits exhibitors</td>
<td>$[1248_{t=28} \ 1248_{t=27} \ 1250_{t=26} \ 1248_{t=25} \ 1242_{t=24} \ 1232_{t=23} \ 1218_{t=22} \ 1200_{t=21} \ ...]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>History profits agents</td>
<td>$[2496_{t=28} \ 2496_{t=27} \ 2500_{t=26} \ 2496_{t=25} \ 2484_{t=24} \ 2464_{t=23} \ 2436_{t=22} \ 2400_{t=21} \ ...]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 13 Summary statistics of a representative run with a myopic monopolist

Table 13 depicts the equilibrium statistics at step 28, two periods after the benchmark model equilibrium and one period after the simulation settles. The first row shows the simulation number, recorded current prices, active agents and corresponding profit which the simulation settles at$^{81}$. Row two and three are the monopolists’ profit for each of the two groups for each previous time period and the last row is just row two and three combined. As highlighted we reach the benchmark model equilibrium at step 26, and after the monopoly increases the prices one more time our satisficing rule is initiated, this is when the monopolist realizes that the previous profit exceeds current profit and stops raising prices toward visitors and exhibitors. This is a direct consequence of the pricing behaviour explained in the setup, with a more aggressive pricing behaviour, the simulations would’ve settled earlier and vice versa.$^{82}$ We can note however that the monopolist always overshoots prices with one monetary unit.

One guess as to why the outcome always settles at the benchmark equilibrium could lie in the fixed expectations and uniform distribution of $\theta$ and $\gamma$. Since our agents have passive beliefs, they do not take into consideration that the active participants of the other side is greater than

![Figure 5 Active visitors, exhibitors and profit over time.](image)

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$^{79}$ Run indicates the simulation, in this case simulation 1 is presented as representative of all the 5000 simulations.

$^{80}$ One step is one time period, step 28 is thus time period 28.

$^{81}$ I let the model run to 50 steps to ensure stability.

$^{82}$ I.e. if we instead programmed it to raise prices for 1.1 each period or 0.9 the time to settlement would change.
what they always believe, which is $x^e = 50$, as case 1. The linear price increases by the monopolist corresponds to the agents leaving the platform in a linear pace as well.

Tracing each step the above three diagrams, figure 5, depicts the dynamics, or lack of dynamics, under uniform assumptions of $\theta$ and $\gamma$ and fixed expectations by the visitors and exhibitors. Although not very exciting in terms of dynamic behaviour the results are in themselves quite interesting. Even though we have alleviated the monopolist of having information regarding the market actors demand functions and expectations, by setting low initial prices through a very simple rule, we approximately reach the benchmark equilibrium every time.

**Case #3.1 Q2. Myopic monopolist and random draw of $\theta$ and $\gamma$**

We run the model with the same random draw setting of $\theta$ and $\gamma$ from case #1.1. Under these circumstances a large variation of equilibrium outcomes are observed. The simulations are shown to settle at profits ranging from 1628 to 3198 by the min- and max values in the table below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runs</td>
<td>10 000</td>
<td>5000.5</td>
<td>2886.896</td>
<td>1</td>
<td>10 000</td>
</tr>
<tr>
<td>Step</td>
<td>10 000</td>
<td>40</td>
<td>0</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Price visitors</td>
<td>10 000</td>
<td>20.17</td>
<td>3.24</td>
<td>10</td>
<td>32</td>
</tr>
<tr>
<td>Price exhibitors</td>
<td>10 000</td>
<td>20.18</td>
<td>3.23</td>
<td>8</td>
<td>31</td>
</tr>
<tr>
<td>Active visitors</td>
<td>10 000</td>
<td>59.38</td>
<td>5.78</td>
<td>41</td>
<td>78</td>
</tr>
<tr>
<td>Active exhibitors</td>
<td>10 000</td>
<td>59.33</td>
<td>5.81</td>
<td>41</td>
<td>79</td>
</tr>
<tr>
<td>Profits$^{83}$</td>
<td>10 000</td>
<td>2369.13</td>
<td>196.41</td>
<td>1620</td>
<td>3185</td>
</tr>
<tr>
<td>max_profits$^{84}$</td>
<td>10 000</td>
<td>2396.112</td>
<td>200.743</td>
<td>1628</td>
<td>3198</td>
</tr>
</tbody>
</table>

*Table 14 Summary statistics at time period 40*

Table 14 shows some descriptive statistics of our simulations at the last step of the simulations. Observing the means of the variables of interest (price, active agents and profit) one may argue that when the distribution is randomly drawn in $\theta$ and $\gamma$ the behaviour of the monopolist becomes quite conservative with the myopic pricing behaviour. On average the number of active visitors and exhibitors succeeds that of the benchmark model of about nine agents on both sides, same pattern are established for prices and profits – both are lower compared to the benchmark model, with a great span as indicated by the standard deviation. Although we cannot be certain of what the profit maximizing conditions in prices and active visitors are when $\theta$ and $\gamma$ are randomly drawn$^{85}$ the fact that the distribution is not linear and that the monopoly has a myopic

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$^{83}$ These are the profits which the simulation settles at, $t_{27}$

$^{84}$ These are the profits the period before the simulation settles, $t_{26}$

$^{85}$ This claim stems from the argument made in using the genetic algorithm for case #1.1.
behaviour and not a maximizing behaviour speaks in favour of the current outcomes not to be considered as the profit maximizing values.\textsuperscript{86}

Convergence toward the stabilized state varies as well, here we can think of convergence as a function of prices.\textsuperscript{87} Since the monopolist increases prices each period until its satisficing condition is fulfilled observing table 14 we can draw the conclusion that every simulation settles somewhere between 10 – 32 periods.

A last qualitative difference of results compared to the uniform distributions of $\theta$ and $\gamma$ is the fact that as the monopolist evaluates and compares its profits from the previous period it does so separately for both visitors and exhibitors, this opens up for different prices toward the two groups. Heterogeneity of prices between the two groups by the monopolist are in line with the results of the genetic algorithm implemented in case #1.1 where we sought to find the profit maximizing prices under random draws of $\theta$ and $\gamma$. While the algorithm in case #1.1 iterates through different prices without any restrictions we have in this model been able to replicate similar pricing behaviour under more narrow assumptions and bounds.

A closer look into the final prices for the plurality of outcomes reveals a bell-curve distribution in figure 6. The X-axis depicts the difference between prices set toward visitors and exhibitors, $p_{vt50} - p_{xt50}$. Differences up to 16 “monetary units” is observed.

Despite the simple nature of our model and agents, the conservative pricing behaviour of the myopic monopolist of our simulations loosely coincide with Nedelscus (2013), where the students were not quite as fierce to set prices equivalent to the benchmark theoretical solution of perfect information.

If we allow ourselves to bring matters to a head we may conclude that our myopic monopoly pricing behaviour conforms to students acting as monopolists in an experimental setting for two-sided industries.

\textsuperscript{86} Preliminary validation of this claim would be to compare the outcome here with the genetic algorithm implemented in case 1.1. An attempt to check the validity of this claim will either way be done in section 7.

\textsuperscript{87} It would be very hard to sift through the data in order to find exactly where each one of the 10 000 simulations are stabilized hence this simple measure used.
Case #4 Intra-heterogeneity in expectations and local interaction

We run each simulation 500 times for 100 steps before stopping. The simulations seem to settle at a stationary distribution, as shown by figure 7. This is most likely due to the changing behavior of how visitors form their beliefs and the introduction of local interaction.

Figure 8 presents the average profits at the last step of each simulation by different vision values. The Y-axis represent profit and the X-axis the different values of “local” vision given to the visitors. What we can observe is that as we grant the agents more vision, the more likely we are to adhere to the equilibrium of the benchmark model with $\pi = 2500$.

The relatively stable movements of each value of vision-setting is most likely due to the fact that our population size is fixed and as is both the exhibition center and the world in which the simulation can be considered to take place. Although some stochastic elements of movement is present in this model compared to previous models a lot is still ordered through our equation-based approach in the movements of our agents. Further the stable outcome of movements present in this model could be explained through the previously mentioned Katz & Shapiro (1985: 425) deductions that there exists a unique equilibrium for each expected fulfilled network size, or in our case, what we observe is the unique equilibrium for each set of expectations.

88 As previously explained in the setup section, this is largely due to the collected data being too vast for the programs to handle. The current simulation has 858 500 observations in total according to the Stata dataset.
89 The simulations have only been run up to distance equaling 16, at 18 it settles for the benchmark equilibrium and any further the model becomes erratic.
90 In the cases concerning naïve- and adaptive expectations only the initializing conditions were randomized, otherwise none of the behavior involved any continuous stochastic elements.
segmented by our agents given the range of vision by NetLogo. We should stress however that we do not observe that the simulation settles, instead we have chaotic fluctuation around the equilibrium.

It should be stated that the monopolist is still able to maximize profits for each value of vision given to the visitors, however since the monopolist is bound by whatever segmented beliefs that are present, they are only able to extract shares of the benchmark model equilibrium whenever the vision of our visitors can be considered narrow. We can determine that this is still in line with the profit maximizing prices as a function of expectations.

Figure 9 depicts the profit maximizing prices for each aggregate average expectations of our visitors given different vision-settings, shown by the Z-axis.

Aligned with the theoretical model, the platform seem to entertain around 50 visitors the closer their vision values resembles the global case distance $\rightarrow 20$ but deviant patterns of relatively fewer visitors is observed with decreasing vision, which is something we should not expect. Comparing the two agent groups, exhibitors who display the same behavior as in case 2.1 converges much faster to its benchmark equilibrium counterpart. If this is the emergence of our expanded continuous induced randomness and heterogeneity, a feature of the given “world” or a bug in the code is unclear at this point and would require further investigation.
6. Discussion
This section will discuss the results in general, potential pitfalls in computer simulations, limitations, potential issues to be vary of and how to deal with verification, validation and replication for agent based models with regard to the experiments of this paper and future explorations.

Discussion of results
The former part of this essay, up to section three, argued for the relatively more dynamic nature of competition in two-sided markets and that because of this the area could potentially benefit from utilizing computer simulations and agent based models as an additional tool. In the latter part of this essay, section four and five, we put this to test by implementing a theoretical model and tried expanding it through both suggestions made by the original authors of said model as well as trying to utilize some of the comparative advantages that agent based simulations brings forth as a tool. In each of our attempts however we have not been successful in presenting results in line with the contention of the arguments presented in the former part of this essay. The analytical equilibrium seemed fairly robust in each of the cases considered, both using initializing conditions invoking stochastic elements and when assumptions were made through empirically valid two-sided dynamics.

Our equation-based approach could be argued not to initiate enough emergence through its interactions between the agents to invoke the dynamic nature of two-sided markets as initially hypothesised. The author have kept the simulations quite close to their theoretical counterpart and whenever/wherever the model have been expanded by invoking a different set of draws in the agents heterogeneous values or their formation of expectations, even these changes have been closely related to assumptions used in theoretical literature in their related areas.

As an example, the changed uniform distribution of \( \theta \) and \( \gamma \) to a random draw (case 1.1) was still kept to be uniform and capped at values between 0 – 1. While arguably validating the analytical derivations of the underlying theoretical model, one could better utilize the agent-based framework by constructing a model describing the empirical outcomes of a particular market to see for what heterogeneous distribution of, for example, \( \theta \) and \( \gamma \) that actually describes this empirical outcome best. Is the uniform distribution a valid assumption for this market? How does an alternative distribution compare? Agent-based models have been critiqued for not being able to produce generalized results; that the models are too detailed to be expanded into other areas, and I think this is the conflict underlying the lack of interesting results in the current paper. The small tweaks in different directions have still kept us in the realm of generality and this is not where the comparative advantage of agent-based modelling lies.

Same principle hold for the models of case 2. The only change in which the Cobweb model assumptions of naïve- and adaptive expectations invoked was that the time toward equilibrium slightly increased, otherwise no qualitative difference emerged from the case 1 with uniform distribution and passive agents.
These choices of keeping close to the theoretical literature have been made largely due to issues of verification and validation which will be detailed in the next section. Only when we considered stochastically induced randomized values of $\theta$ and $\gamma$ did the results confine with what we may observe empirically, of different network sizes between the two agent groups and different prices toward the two groups.

Even considering the pricing-behaviour of the myopic monopolist of case 3 with a uniform distribution of $\theta$ and $\gamma$ and fixed expectations by our agents could be said to be too simple and easily derived using pen and paper, hence the uninteresting results. Introducing more factors to be evaluated for the monopolist when setting prices would of course increase the complexity of the dynamics of this model and make it more interesting and accurate. I.e. making it more aggressive by exploring new prices if the same profit is maintained for a certain amount of time periods, introduce transaction prices\(^{91}\) or introducing an additional condition of the monopolist aiming for a minimum amount of active agents to always be present on the platform. Introducing costs to be considered, or that exhibitors occupy different amount of space on the fair and pays a price accordingly would also be more complex ways to consider introducing. For a more thorough overview of the potential to model dynamic pricing Boer (2015) provides an astonishing compilation of practices used.

In a last attempt to better utilize the comparative advantage of agent-based models we tried to increase the heterogeneity of our agents in two ways, giving each agent their own stochastically induced list of expectations as well as updating the expectations as a function of their vision, where a NetLogo primitive was used. We could observe in this case how simply limiting the available information, or introducing local interaction in other terms, for the agents changes the profit extraction for the monopolist only to extract shares of the benchmark model profit. When we move from general theoretical models to empirical application, this is obviously an important question and most likely would depend on the market in question studied; what factors are important in the formation of beliefs in this market? From what sources do agents get their information influencing their expectations and in length the likelihood of adoption themselves? What can the platform providers do to induce more positive expectations on part of the relevant parties who might find value in adopting the platform? The next natural step would be to endogenously program expectations to be a function of relevant characteristics, environmental considerations and important factors with a specific market/industry in mind.

In summary, the fact that the cases considered did not move too far from the theoretical literature should’ve been telling of the lack of expected dynamic results. By adhering to these restrictions the full potential use of agent-based models have not been utilized in the current paper. In essence there is still only a small part of heterogeneity invoked in part of the agents in the model. Symmetrical demand functions, uniform distributions of their heterogeneity parameters and same

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\(^{91}\) To this point and the underlying theoretical model used have only considered membership prices. This is also something brought forth in GWs final remarks.
beliefs does not bring us as far away from the often theoretical literatures assumptions of the representative consumer which then is aggregated to predict market outcomes.

Unfortunately I have not managed to depart from the stable system approach, the current essay should perhaps be seen as an exploratory exercise in exploring theoretical studies with an agent based approach. With more time and experience in this approach one would be able to better utilize the opportunities computer simulations offers and supply the discipline of economics useful dynamic models moving away from the stable system approach in order to help explain real world economic phenomena and issues.

Indicative of this last paragraph would be the review of agent-based models in section 2, especially interesting are the cases where simulation can be used to evaluate counterfactual scenarios. This is something the author believes to be of potential value for any agency which has in its role to, when necessary, intervene in a particular market. Given that data have been gathered and a model created which correctly replicates the descriptive market under investigation how would some induced rule invoked by the agency change the dynamics of the market? Does the rule have its intended effect, or do other behaviour emerge which was unforeseen at first hand?

**Validation, verification and replication**

Wilensky & Rand (2015: 311 - 312) make three types of distinctions in the process of evaluating a model. Verification to ensure that the implemented model corresponds to the intended model, these are the measures and steps taken in the implementation of code and to understand the interaction of the agents in the model and ensure their reliability, to understand why the reported outcomes are what they are. Validation is the process of the *output* of the model describing some real world phenomena the researcher is interested in, do the two align? And lastly replication, similar to traditional empirical economic research, we require that other researchers have the necessary information and tools at their disposal to be able to replicate the findings.

In this section we will focus mainly on verification. As for validation, given that the assumptions used have not moved too far from the theoretical literature, it would make more sense to compare our outcomes to their theoretical counterparts rather than actual empirically observed two-sided market outcomes.

**Verification**

Verification deals with the issue of debugging, or in other words, eliminating “bugs” from the code (Ibid: 313). The issue of verification and in length the reliability of results presented in this paper should not be downplayed. The author would like to point out several sources of

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92 For the author, not the discipline as a whole. Computer simulations and agent based models have been utilized for some time, although perhaps not to the extent corresponding to its potential value.

93 Perhaps one could crudely view verification as internal validity and validation as external validity in comparing the concepts to econometrics

94 For the interested any of the agent-based simulation papers presented in this essays are great examples of agent behaviours modelled for specific two-sided industries.
uncertainty in this regard and why caution should be invoked in extrapolating the presented results or even taking them at face value.

First source of uncertainty stems from the lack of programming experience of the author and the lack of familiarity of the program. As previously declared the process of learning the programming language have been parallel to this essay for the author, to the degree possible caution has been taken in several ways when implementing code. While Wilensky & Rand (313 - 315) exemplifies the verification process of having one modeller, knowledgeable of the program, and a researcher, with the intended model to be implemented, the author of this paper have had to play both parts. Evident of some of the pitfalls Wilensky & Rand (2015) highlight are for example exactly how different primitives function in the program. Which primitives would best suit the researcher? Or if the primitives do not satisfy the criterion the researcher sets, how can this be programmed? It is not unreasonable that some of these pitfalls might’ve been present in the current paper and have gone unnoticed by the author. In addition to this an additional step of insurance could’ve been to utilize the process of writing pseudo-code (Ibid: 314) for additional eyes in order to assess the plausibility of the said implementation of code at an earlier stage of this project.

As for the actual code and models, to the extent possible, measures have been taken to ensure that the simulations are working as intended. These measures have been separating the different models into independent files in order to avoid errors and potential pitfalls of code interacting where they shouldn’t, segmentation of code into separate small procedures, taking advantage of the low number of agents and the graphical output in NetLogo in order to trace that the agents are in fact infused with the correct numerical values of their heterogenetic measures as well as the fact that they actually do move back and forth in line with their utility functions and so forth. These safety measures are however only conducted by the author and is not entirely sufficient, as this most obviously are a part of the process itself of building agent-based models.

In light of these conditions the code reflecting the four models have been simple and perhaps not as efficient as it could be. Here efficient can be thought of in two ways, computational efficiency and the ability to extract further measures of interest of the models. Computational efficiency would be in the way of understanding if the implemented code resembles the most effective way of coding the aimed behaviour and interaction, which again can only be assessed with more experience of the program or someone knowledgeable of the program. The latter more related to

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95 And being an amateur in both does not bolster any more confidence.
96 Primitives are functions provided by the program itself which can be used. I.e. one does not have to implement code in order to do simple addition. This is provided by the program.
97 Pseudo-code is when you write the code in an algorithmic structure, acting as a map which can be read by a non-programmer with relative ease.
98 A simple example of this would be to program the visitors and exhibitors to change colour whenever their utility are strictly positive, in this way the graphical output can reveal whether in fact all agents with positive utility have bought access to the platform. This is what Wilensky & Rand (2015: 317) calls “unit testing”.

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economics would be to think of how to elaborate the code in order to extract further welfare measures, such as visitor- and exhibitor surplus in the different cases considered.

One issue that has remained unsolved in the end of the essay that is most likely due to potential programming issues are the 22 cases where the network is breaking down in #2.2 and $\lambda = 1$. Although the simulations are initiated and the early time periods follow the same pattern as in #2.1, with one agent group expecting 0 at the start, instead of settling for the 2-cycle $\pi_{1250}$ equilibrium, the network breaks down instead. The additional complexity of code necessary to ensure the correct workings of the model when $\lambda < 1$ might here interfere with the case when $\lambda = 1$. Fortunately the simplistic nature of the model, the general lack of sensitivity of initial conditions and the cases being very much separated/isolated from each other, this issue should remain contained only for #2.2.  

Model #4 should be stated evolved quite late in the course of the time period set for this project and although no visible errors have revealed themselves the code has not been under the same scrutiny as the three former models. Meyer (2012:74-75) also outlines many of the above mentioned methods of verification and with the increased stochasticity of model 4, Ibid (2012:74 - 76) adds that stochastic models should be run more than once with extensive logging on how the agents act over time in the simulation in the verification process.

Going by the literature, the best verification resource would in the end be valid scrutiny from other researchers evaluating the code and implementing the model in another programming language, in other words replication (Ibid: 337 – 340).

Validation

Case #1

Moving toward validation Wilensky & Rand (2015: 329) make a distinction between micro- and macro validation. The former being validation at the level of agents acting according to their target counterpart, be it empirical behaviour or behaviour set by a theoretical model, and the latter being the aggregate outcome of all the interactions of the model. The lack of heterogeneity in our agents and the overall equation-based approach of our simulations in this paper have less need for the former type of validation and thus should be more concerned with the latter type (Ibid : 331). In a sense we have not been able to move too far away from the representative agent of neoclassical modelling.

Focusing on the latter type of validation in case 1 we were able to replicate proposition 1 of the analytical outcome of GW, we did not however manage to replicate proposition 2 but instead got

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99 So in considering the outcomes when agents have naïve expectations, the outcomes related to model #2.1 should take precedence over #2.2 when $\lambda = 1$

100 Reaching the end of the essay period it may be added that the tutor of said author of this essay managed to implement the model of naïve expectations using Stata. The implemented code and the short comparison of results from the simulations seem to coincide with what was done here in NetLogo, although no elaborate systemic comparison has been made between the code and models.
an alternative 2-type cycle with \( \pi = 1250 \) instead. Two potential causes come to mind as to why we were unsuccessful in reaching proposition 2. One would be that proposition 2 is by its nature an unstable equilibrium point which to my understanding can only be reached through a unique set of starting positions in expectations or two, the programming intervening\(^{101}\), or the combination of the two.

In the following cases where we introduced different types of behaviours on part of both the monopolist as well as the agents, the analytical outcome was kept fairly robust as well. The aggregate outcomes of the subsequent models could be seen as auxiliary evidence that we have managed to replicate the first model correctly. And perhaps most convincingly, one can calculate the equilibrium outcome by hand given the population size and set of assumptions used in the simulation.

In length we also addressed an alteration of their suggestion of attempting a different draw of \( \theta \) and \( \gamma \) to see if their analytical outcome held qualitatively. Using the complementary tool BehaviorSearch\(^{102}\) a genetic algorithm was used to find the profit-maximizing prices with \( \theta \) and \( \gamma \) randomly drawn. The suggestion Wilensky & Rand (2015:339) makes regarding the use of search algorithms\(^{103}\) and validation is to either program the iterative process oneself if possible, or use another type of search algorithm and compare the outcomes. As the former approach have proved to be too difficult we take advantage of BehaviorSearch again and another type of search algorithm to see whether the results correspond to the one used in the result section.\(^{104}\) However it should be said that despite letting the algorithm run for the length specified one cannot be fully confident that the profit maximizing prices found are in fact the correct ones. The longer you let the algorithm run, the larger is the confidence that the found combination is in fact the best one. It can be argued that the number of simulations and the length of search in each simulation run in our case has been insufficient.\(^{105}\) Limited computing power has certainly been a bottleneck for the current project\(^{106}\)

\(^{101}\) If our agents only move when their utility is strictly positive and prices are set as a function of expectations, then any agent group expecting zero participants from the other side would not adopt the platform even when their price is set to zero, given the strict condition.
\(^{102}\) http://www.behaviorsearch.org/
\(^{103}\) There are several search algorithms in computer science, of which a handful is available in the Behavioursearch toolkit. The two used in the validation process here and forthcoming in case 3 is random search and the genetic algorithm.
\(^{104}\) We kill two birds with one stone in this regard by attempting this validation along with the claim that the myopic monopolist was not profit-maximizing for case #3 to come.
\(^{105}\) Some issues of running the algorithm on the personal computer of the author has also been present during the implementation of the algorithm, hence the few runs for the algorithm. Letting it run for several thousand runs and letting it search for days would be more optimal.
\(^{106}\) Meyer (2012: 174) under the testing phase of his model mentions testing “several hundred thousand runs” which took more than a week on a personal computer and how that was considered a bottleneck for his research project. The study of Huoutari et.al (2015: 21) presented in section 2 declares simulating several million models in total.
Case #2

The validation process of the simulations when we have considered active beliefs becomes an issue as we have neither aspired to describe a specific empirically two-sided market outcome nor have a theoretical model to lean against compared to when we replicated the equilibrium outcome under passive beliefs. The active beliefs implemented in the models related to case 2 are borrowed from the Cobweb literature, thus the author sees as his option to assess the plausibility of our outcomes with that of studies using the approach of studying belief formations in different areas (Sonnemans et.al, 2003; Hommes & Sorger, 1998; Chen & Yeh, 1996) under the similar Cobweb framework. Albeit all being more technical and elaborate the listed studies have been used in evaluating and interpreting our own results. In general the Cobweb model seem to be able to produce three different outcomes depending on the given parameters of a model and the relationship between supply and demand, these are convergence to a steady state, divergence and stable cycles. Using these general outcomes and comparing it to our own results all three seem to be present. In almost all simulations considering #2.1 - #2.2 we have convergence, which would indicate that the relationship between the three interacting agents in our model can be considered to have fairly stable initializing conditions. The only parameter value which induced divergence and cycles were whenever any of the initialising conditions was stochastically chosen to be zero.

Case #3

Our third model inherits the same kind of validation issues as case 2. Although we have established a different kind of price-setting rule by the monopolist which could be argued to be less knowledge intensive, in order for us to validate the output the agents have again been given fixed expectations. As previously, we know what the profit-maximizing prices are, and that the monopolist does not. Our passive beliefs on part of our agents allows the model to settle for the equilibrium of the benchmark model if all the criteria are fulfilled. We have thus put more weight on verification rather than validation, ensuring the model is working as intended rather than to harness interesting results from more active agents.

One proposed route could be to compare the outcomes of this kind of price-setting rule with that of the study inspiring the implementation of this kind of myopic monopolist behaviour in the current paper (Heinrich & Gräbner, 2015). The level of complexity between the two models are

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107 The steady state being a stable price in this case, where supply and demand manages to meet.
108 Divergence indicates that supply and demand move further away from each other and if there are no restrictions built in, they keep diverge to infinity.
109 This is when a set of prices go back and forth continuously.
110 I.e. we have programmed the initializing states to be in the span of regularity, leading to stable outcomes.
111 That the price-setting behaviour finds it reasonable to settle for the benchmark outcome.
112 Ex-post, it is evident that further explorations could be made with more active beliefs since we managed to reach the benchmark-equilibrium even with this type of endogenous pricing-behaviour, however it was not certain the author would reach this point at the start of this project.
however considered too great for any fruitful comparison and potential validation for our results.\footnote{They do consider a competitive environment of multiple platforms with different price-setting rules and different levels of knowledge of the market in question.}

In the result section under the assumption of a random draw of $\theta$ and $\gamma$ and myopic monopolist I argued why the settled prices in this case probably did not adhere to the profit-maximizing prices. The route we can try to take in order to assess whether this is correct or not is to again make use of BehaviorSearch. Comparing the results of case 3.1 Q.2, and running the model through different algorithms maximizing profit with the algorithm toolkit, one approach may be to average the profit, prices and active agents to see how and if they differ. To current state, this is the best solution found.

<table>
<thead>
<tr>
<th>Genetic algorithm. (From case 1.1 Q.2.)</th>
<th>Observations</th>
<th>Mean</th>
<th>Std.dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price visitors</td>
<td>200</td>
<td>25.47</td>
<td>3.58</td>
<td>17</td>
<td>34</td>
</tr>
<tr>
<td>Price exhibitors</td>
<td>200</td>
<td>25.07</td>
<td>3.54</td>
<td>16</td>
<td>34</td>
</tr>
<tr>
<td>Profit</td>
<td>200</td>
<td>2498.93</td>
<td>49.75</td>
<td>2367.2</td>
<td>2658.2</td>
</tr>
</tbody>
</table>

*Table 15 Summary statistics with the genetic algorithm*

<table>
<thead>
<tr>
<th>Random search</th>
<th>Observations</th>
<th>Mean</th>
<th>Std.dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price visitors</td>
<td>1000</td>
<td>25.46</td>
<td>3.86</td>
<td>14.4</td>
<td>36.7</td>
</tr>
<tr>
<td>Price exhibitors</td>
<td>1000</td>
<td>25.32</td>
<td>3.76</td>
<td>14.6</td>
<td>36.9</td>
</tr>
<tr>
<td>Profit</td>
<td>1000</td>
<td>2534.11</td>
<td>62.66</td>
<td>2295.4</td>
<td>2747.7</td>
</tr>
</tbody>
</table>

*Table 16 Summary statistics with the random search algorithm*

<table>
<thead>
<tr>
<th>Case 3 #Q2 (From our result section)</th>
<th>Observations</th>
<th>Mean</th>
<th>Std.dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price visitors</td>
<td>10000</td>
<td>20.17</td>
<td>3.24</td>
<td>10</td>
<td>32</td>
</tr>
<tr>
<td>Price exhibitors</td>
<td>10000</td>
<td>20.18</td>
<td>3.23</td>
<td>8</td>
<td>31</td>
</tr>
<tr>
<td>Profit</td>
<td>10000</td>
<td>2369</td>
<td>196.41</td>
<td>1620</td>
<td>3185</td>
</tr>
</tbody>
</table>

*Table 17 Summary statistics with the myopic monopolist of case #3.1*

The comparison of mean prices between the three methods could shed some light on whether the myopic monopolist managed to set the profit maximizing prices on average. Comparing the average prices of case 3 with that of the two search-algorithms it does seem like our initial explanation holds, when the distribution of $\theta$ and $\gamma$ is randomly drawn the pricing behaviour implemented does not manage to reach the height of profits compared to the neoclassical assumption of perfect information. It should be noted however that the variation in simulation outcomes is much larger for the myopic provider when compared to the algorithms, which in this case can be considered our rational provider.

Our previous discussions regarding the relatively low number of times the algorithm has been allowed to search and for how many observations should be mentioned is still present.
Case #4

In model 4, the increased heterogeneity and chaotic fluctuation surrounding the best prices the monopoly can extract, given expectations, a comparison could perhaps be found in Velden (2001: 28 – 29) where a similar pattern is presented under heterogeneous expectations. Velden (2001) presents an example of how a simple heterogeneous environment can produce chaotic price fluctuations around the rational equilibrium steady state (Ibid: 27). In the presented example there are two types of producers who alternates between two set of forecasting rules, depending on previous performance, and goes to show how the competition between the two rules produces the chaotic pattern in pricing. In validating case 4, this is the best simulation counterpart found. Our counterpart inducing the chaotic fluctuation in model 4 would be the local interaction by the visitors, and to some extent the adaption by exhibitors combined with the price-setting behavior of our monopolist always trying to maximize its profits by using the average of our agents expectations.

7. Summary

Departing our explorations in this essay with replicating the analytical solution of GW in an agent based framework was successful for proposition 1, however we were not able to produce proposition 2. The following incremental departures of each subsequent model have proved the analytical equilibrium to be robust for most of the changes invoked as well.

A random draw of $\theta$ and $\gamma$ with a perfectly informed monopolist was found to be a fairly regular alternative to the uniform distribution upholding the qualitative solution of the analytical equilibrium with an infinite population although each unique random draw of $\theta$ and $\gamma$ invoked heterogeneous prices and different network sizes between the two groups with its own unique equilibrium.

The stochastic starting points of our naïve- and adaptive agents in case 2 proved robust convergence toward the analytical equilibrium despite the large variation in their initializing values of their previous beliefs. Only when initialized with expecting zero agents from the other side did the benchmark equilibrium alter to breaking down or to produce 2-way cycles.

In model three with a myopic monopolist and passive agents the simulation outcomes proved again to be robust for the analytical equilibrium. Under the same pricing policy and a random draw of $\theta$ and $\gamma$ we found that the myopic monopolist tended to be more conservative and do worse on average than the platform provider with perfect information. For each individual simulation run a wide range of outcomes was observed however, some managing to do far better and some far worse than the rational monopolist.

In model four introducing unique previous expectations for each agent, locally formed expectations by visitors and local interaction did chaotic fluctuations surrounding the profit-maximizing equilibrium develop.
The findings of the simulations in this paper should not be extrapolated any further than what the assumptions and initializing conditions allow for. Acknowledging the simplified nature of our two-sided market and the limitations I argue, conservatively, that the contribution are twofold.

The ease of changing assumptions, the initializing conditions and altering theoretical models in general with this framework of agent-based modelling allows for richer theoretical explorations, which can complement traditional theoretical studies focusing on closed form solutions.

Second, in some instances we managed to utilize the comparative advantage of the agent-based framework by invoking local interaction and endogenous bounded pricing-behaviour. In this regard the current exploration have added a small fragment to the cumulative library studying economics through agent based models.
8. References

Ardolino, Marco, Nicola Saccani, and Marco Perona. "The analysis of multisided platforms: Results from a literature review."


**Software**


**Websites**


9. Appendix

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Theoretical functions and simulation-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_v$</td>
<td>Utility of visitors</td>
<td>$U_v = \theta x^e - p_v$</td>
</tr>
<tr>
<td>$U_x$</td>
<td>Utility of exhibitors</td>
<td>$U_x = \gamma v^e - p_x$</td>
</tr>
<tr>
<td>$N_v$</td>
<td>Total number of visitors</td>
<td>100</td>
</tr>
<tr>
<td>$N_x$</td>
<td>Total number of exhibitors</td>
<td>100</td>
</tr>
<tr>
<td>$x^e = x^{114}$</td>
<td>Expectations visitors have about the number of exhibitors</td>
<td>$D^v = 1 - \frac{p_v}{x^e}$</td>
</tr>
<tr>
<td>$v^e = v$</td>
<td>Expectations exhibitors have about the number of visitors</td>
<td>$D^x = 1 - \frac{p_x}{v^e}$</td>
</tr>
<tr>
<td>$P_v$</td>
<td>Price toward visitors</td>
<td>$P_v = \frac{x^e}{2}$</td>
</tr>
<tr>
<td>$P_x$</td>
<td>Price toward exhibitors</td>
<td>$P_x = \frac{v^e}{2}$</td>
</tr>
<tr>
<td>$\theta_l$</td>
<td>Heterogeneous variable measuring the value a visitor derives from potential trades in the fair</td>
<td>$u[0,1]$</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>Heterogeneous variable measuring the value an exhibitor derives from having a stand in the fair.</td>
<td>$u[0,1]$</td>
</tr>
</tbody>
</table>

Table 18 Summary of initialized parameters and functions

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$^{114}$ $x$ and $v$ are the corresponding active agents on each side under the assumption of fulfilled expectations.
Figure 11 Graphical output produced in NetLogo running case #1
Figure 12 Graphical output produced in NetLogo running case #4